MAT598 - Additional Problem Set 04

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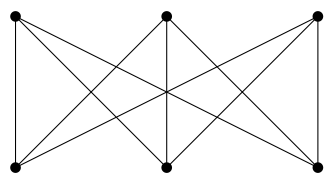
1. Let $g \geq 2$ and let X be the surface of genus g. Let $Y = S^2 \vee \underbrace{S^1 \vee \cdots \vee S^1}_{2g}$.

- **a.** Show that $\pi_1(X) \neq \pi_1(Y)$.
- **b.** Show that the abelianization of $\pi_1(X)$ is isomorphic to the abelianization of $\pi_1(Y)$.

2. Let K be the graph formed from 6 vertices and 9 edges as shown below, and let X be formed by attaching a 2-cell along each loop formed by a cycle of four edges. Show that $\pi_1(X) = 0$.



3. *[May 2015]* Let X denote the topological space obtained by gluing the meridian of a torus to a longitude of another torus. Find a presentation for $\pi_1(X)$.

4. *[August 2015]* Let X denote the topological space obtained by gluing the boundary of a Möbius strip to a meridian of a torus. Find a presentation for $\pi_1(X)$.

5. Let X be the disk, annulus, or Möbius band, and let $\partial X \subseteq X$ be its boundary circle or circles.

- **a.** For $x \in X$, show that the inclusion $X - \{x\} \hookrightarrow X$ induces an isomorphism on $\pi_1(X)$ iff $x \in \partial X$.
- **b.** If Y is also a disk, annulus, or Möbius band, show that a homeomorphism $f : X \to Y$ restricts to a homeomorphism $\partial X \to \partial Y$.
- **c.** Deduce that the Möbius band is not homeomorphic to an annulus.

6. The *mapping torus* T_f of a map $f : X \to X$ is the quotient $X \times I/(x, 0) \sim (f(x), 1)$. In the case $X = S^1 \vee S^1$ with f basepoint-preserving, compute a presentation for $\pi_1(T_f)$ in terms of the induced map $f_* : \pi_1(X) \to \pi_1(X)$. Do the same when $X = S^1 \times S^1$. [One way to do this is to regard T_f as built from $X \vee S^1$ by attaching cells.]