MAT502 - Additional Problem Set 03

Joseph Wells
Arizona State University

Spring 2017

1. Suppose $M \subseteq \mathbb{R}^n$ is an embedded m-dimensional submanifold, and let $UM \subseteq T\mathbb{R}^n$ be the set of all unit tangent vectors to M:

$$UM = \{(x,v) \in T\mathbb{R}^n \; : \; x \in M, v \in T_x M, |v| = 1\}.$$

It is called the **unit tangent bundle of M.** Prove that UM is an embedded $(2m-1)$-dimensional submanifold of $T\mathbb{R}^n \cong \mathbb{R}^n \times \mathbb{R}^n$.

2. For each $a \in \mathbb{R}$, let M_a be the subset of $\mathbb{R}^2$ defined by

$$M_a = \{(x,y) \; : \; y^2 = x(x-1)(x-a)\}.$$

For which values of a is M_a and embedded submanifold of $\mathbb{R}^2$? For which values can M_a be given a topology and a smooth structure making it into an immersed submanifold?

3. Show by example that an immersed submanifold $S \subseteq M$ might have more than one topology and smooth structure with respect to which it is an immersed submanifold.