Recitation 01: Introduction to Vectors

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For the policies, lectures notes, and homework assignments, please visit my website:

`http://www.joedub.net`

Example. Which has a greater horizontal component, a 100 N force directed at an angle of 60° above the horizontal, or a 60 N force directed at an angle of 30° above the horizontal?

Recall that, for a given vector $\mathbf{V}$ and an angle θ above the horizontal, we can write $\mathbf{V} = \langle |\mathbf{V}| \cos\theta, |\mathbf{V}| \sin\theta \rangle$. As such, let $\mathbf{U}$ be the vector representing 100 N at an angle of 60° above the horizontal, and $\mathbf{V}$ be the vector representing 100 N at an angle of 30° above the horizontal. We then have that

$$\mathbf{U} = \langle 100\cos 60°, 100\cos 60° \rangle \text{ N} = \langle 50, 50\sqrt{3} \rangle \text{ N}$$
$$\mathbf{V} = \langle 60\cos 30°, 60\cos 30° \rangle \text{ N} = \langle 30\sqrt{3}, 30 \rangle \text{ N}.$$

Since the first component represents the horizontal component and $30\sqrt{3} > 50$, vector $\mathbf{V}$ has the greater horizontal component.

Example. Suppose that there are two forces acting on a sky diver: gravity at 180 lb down and air resistance. If the net force is 20 lb down and 20 lb to the left, what is the force of air resistance acting on the sky diver?

Solution (sketch)

Let $\mathbf{G} = \langle 0, -180 \rangle$ be the gravity force vector and $\mathbf{A} = \langle x, y \rangle$ be the air resistance force vector. Then since the net force is given by the vector $\langle -20, -20 \rangle$, we have

$$\begin{aligned} \mathbf{G} + \mathbf{A} &= \langle -20, -20 \rangle \\ \Rightarrow \quad \mathbf{A} &= \langle -20, -20 \rangle - \mathbf{G} \end{aligned}$$

From here you should be able to solve for $\mathbf{A} = \langle \_\_\_\_\_, \_\_\_\_\_ \rangle$.

Example. The thrust of an airplane's engines produces a speed of 600 mph in still air. The wind velocity is given by $\langle -30, 60 \rangle$ mph. In what direction should the airplane head to fly due west?

Solution (sketch)

Let $\mathbf{W} = \langle -30, 60 \rangle$ mph be the wind's velocity vector and $\mathbf{P} = \langle p_1, p_2 \rangle$ mph be the plane's velocity vector. We want

$$\mathbf{P} + \mathbf{W} = \langle p_1 - 30, p_2 + 60 \rangle = \langle x, 0 \rangle, \quad \text{where } x < 0.$$

This implies that $p_2 = -60$. Using $600 = |P| = \sqrt{p_1^2 + p_2^2} = \sqrt{p_1^2 + 60^2}$, solve for p_1. Note that you will have both a positive and negative square root; use the one that makes the plane move due west (the other makes the plane move due east).

Example. Use vectors to determine whether the points $(0, 1, 1)$, $(2, 4, 2)$, and $(3, 1, 4)$ form an equilateral triangle.

Solution (sketch)

Recall that the vector between two points $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ is given by $\overrightarrow{XY} = \langle y_1 - x_1, y_2 - x_2, y_3 - x_3 \rangle$. That being said, label the points

$$P = (0, 1, 1), \quad Q = (2, 4, 2), \quad \text{and} \quad R = (3, 1, 4).$$

Then

$$\begin{aligned}
\overrightarrow{PQ} &= \langle 2 - 0, 4 - 1, 2 - 1 \rangle = \langle 2, 3, 1 \rangle \\
\overrightarrow{QR} &= \langle 3 - 1, 1 - 4, 4 - 2 \rangle = \langle 2, -3, 2 \rangle \\
\overrightarrow{RP} &= \langle 0 - 3, 1 - 1, 1 - 4 \rangle = \langle -3, 0, -3 \rangle.
\end{aligned}$$

To see if PQR is an equilateral triangle, one just has to check if $|\overrightarrow{PQ}| = |\overrightarrow{QR}| = |\overrightarrow{RP}|$.

Assignment

Worksheet 01:

https://mathpost.asu.edu/~wells/math/teaching/mat272_spring2015/homework01.pdf

As always, you may work in groups, but every member must individually submit a homework assignment.