Name:__________________________

§12.7 Tangent Planes and Linear Approximation

1. Find equations of the tangent plane and normal line to the surface $z = \dfrac{4x}{y}$ at the point $(1, 2, 2)$.

2. Compute the linear approximation of the function $f(x, y) = \sin(x)\cos(y)$ at the point $(0, \pi)$.

3. Compute the linear approximation of the function $f(w, x, y, z) = \cos(xyz) - w^3x^2$ at the point $(2, -1, 4, 0)$.

4. Find the total differential of $f(x, y) = ye^x + \sin(x)$.

5. For the given function, show that $f_x(0,0)$ and $f_y(0,0)$ both exist, but that the function is not differentiable at $(0,0)$: $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

6. Let S be the surface defined parametrically by $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$. Define $\mathbf{r}_u(u,v) = \langle x_u(u,v), y_u(u,v), z_u(u,v) \rangle$ and $\mathbf{r}_v(u,v) = \langle x_v(u,v), y_v(u,v), z_v(u,v) \rangle$. Show that $\mathbf{r}_u \times \mathbf{r}_v$ is normal to the tangent plane at the point $(x(u,v), y(u,v), z(u,v))$.

§12.8 Maximum/Minimum Problems

7. Locate all critical points and classify them: $f(x, y) = 2x^2 + y^3 - x^2y - 3y$.

8. Find the absolute extrema of the function $f(x, y) = x^2 + y^2 - 4xy$ on the region bounded by $y = x$, $y = -3$, and $x = 3$.

9. Find the absolute extrema of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ on the region bounded by $y = x$, $y = 3$, and $x = 0$.

10. Find the maximum of $z = x^2 + y^2$ on the square with $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

11. Find the closest point on the cone $z = x^2 + y^2$ to the point $(2, -3, 0)$.

12. The **Hardy-Weinberg law** of genetics describes the relationship between the proportions of different genes in populations. Suppose that a certain gene has three types (e.g. blood types of A, B, and O). If the three types of proportions p, q, and r, respectively, in the population, then the Hardy-Weinberg law states that the proportion of people who carry two different types of genes equals $f(p, q, r) = 2pq + 2pr + 2qr$ where $p + q + r = 1$. Show that the maximum value of $f(p, q, r)$ is $\frac{2}{3}$.