Name:__________________________

§11.9 Curvature and Normal Vectors

1. Find the unit tangent vector to the curve at the indicated points: $\mathbf{r}(t) = \langle 3t, \cos(2t), \sin(2t)\rangle$, at $t = 0$, $t = -\pi$, $t = \pi$.

2. Find the curvature of the given curve at the given point: $\mathbf{r}(t) = \langle 2, \sin(\pi t), \ln(t)\rangle$ at $t = 1$.

3. Find any points of maximum or minimum curvature: $\mathbf{r}(t) = \langle 2\cos(t), 3\sin(t)\rangle$.

4. Find the curvature of the circular helix: $\mathbf{r}(t) = \langle a\cos(t), a\sin(t), bt\rangle$ (here, a and b are arbitrary constants).

5. Find the curvature of the polar curve at the indicated points: $r = \sin(3\theta)$, at $\theta = 0$ and $\theta = \frac{\pi}{6}$

6. For the logarithmic spiral $r = ae^{b\theta}$, show that the curvature equals $\kappa = \dfrac{e^{-b\theta}}{a\sqrt{1+b^2}}$. Show that as $b \to 0$, the spiral approaches a circle.

§12.1 Planes and Surfaces

7. Find the distance between the point $(1,3,0)$ and the plane $3x + y - 5z = 2$.

8. Suppose that $\langle 2,1,3\rangle$ is a normal vector for a plane containing the point $(2,-3,4)$. Show that an equation of the plane is $2x + y + 3z = 13$. Explain why another normal vector for this plane is $\langle -4,-2,-6\rangle$. Use this normal vector to find an equation of the plane and show that the equation reduces to the same equation, $2x + y + 3z = 13$.

9. Find an equation of the plane containing the lines

$$\ell_1 = \begin{cases} x = 1 - t \\ y = 2 + 3t \\ z = 2t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 1 - s \\ y = 5 \\ z = 4 - 2s \end{cases}.$$

For problems 4–6, sketch the appropriate traces, and then identify the surface:

10. $9x^2 + y^2 + 9z^2 = 9$

11. $9x^2 + z^2 = 9$

12. $x + y^2 + z^2 = 2$