Name:____________________________

§11.7 Motion in Space

1. Find the force acting on an object of mass 10 kg with the given position function (units of meters and seconds): $\mathbf{r}(t) = \langle 3\cos(4t), 2\sin(5t)\rangle$.

2. A projectile is fired with initial speed $v_0 = 100\,\text{ft/s}$ from a height of $h = 0\,\text{ft}$ at an angle $\theta = \frac{\pi}{6}$ above the horizontal. Assuming that the only force acting on the object is gravity, find the maximum altitude, horizontal range, and speed at impact.

3. Beginning with Newton's second law of motion, derive the equations of motion for a projectile fired from altitude h above the ground at an angle θ to the horizontal and with initial speed equal to v_0.

4. For the general projectile of Exercise 3, with $h = 0$,

(a) show that the horizontal range is $\dfrac{v_0^2 \sin(2\theta)}{g}$ and

(b) find the angle that produces the maximum horizontal range.

5. A baseball pitcher throws a pitch horizontally from a height of 6 ft with an initial speed of 130 ft/s. Find a vector-valued function describing the position of the ball t seconds after release. If home plate is 60 feet away, how high is the ball when it crosses home plate?

6. A tennis serve is struck horizontall from a height of 8 ft with initial speed 120 ft/s. For the serve to count (be "in"), it must clear a net that is 39 feet away and 3 feet high and must land before the service line 60 feet away. Find a vector function for the position of the ball and determine whether this serve is in or out.

§11.8 Length of Curves

7. Find the length of the following curve: $\mathbf{r}(t) = \langle 4\cos t, 3\cos t, 5\sin t\rangle$, $0 \le t \le 2\pi$.

8. For the following trajectory, find the speed associated with the trajectory and then find the length of the trajectory on the given interval: $\mathbf{r}(t) = \langle e^t \cos t, e^t \cos t, e^t\rangle$, $0 \le t \le \ln(2)$.

9. Use a calculator to approximate the length of the following curve (to three decimal places): $\mathbf{r}(t) = \langle t, 4t^2, 10\rangle$, $-2 \le t \le 2$.

10. Find the length of the following spiral: $r = 2e^{2\theta}$, $0 \le \theta \le \ln(8)$.

11. Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter: $\mathbf{r}(t) = \langle 5\cos t, 3\sin t, 4\sin t\rangle$, $0 \le t \le \pi$.

12. Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter: $\mathbf{r}(t) = \langle e^t, e^t, e^t\rangle$, $t \ge 0$.