Name:_____

§11.5 Lines and Curves in Space

1. Find the limit if it exists. $\lim_{t\to 5}\left\langle e^t, t^2-5, \frac{t+6}{t-5}\right\rangle$

2. Determine all values of t at which the given vector-valued function is continuous.
   $\mathbf{r}(t) = \left\langle \ln(t-2), t+7, \frac{t-9}{t+9}\right\rangle$

3. Find parametric equations of the line through $(-2, 7, 6)$ parallel to $\langle 3, 6, 7\rangle$.

4. Find the parametric equations of the line through $(-1, -7, -9)$ and $(1, -9, 6)$.

5. Find the parametric equations of the line through $(8, 6, 9)$ and perpendicular to both $\langle 6, 0, -1\rangle$ and $\langle -1, -3, 7\rangle$.

6. Determine if the lines are parallel, skew, or intersect.

$$\ell_1 \begin{cases} x = 6 + 2t \\ y = -4 - 6t \\ z = -4 + 6t \end{cases} \quad \text{and} \quad \ell_2 \begin{cases} x = 8 - s \\ y = -10 + 3s \\ z = 2 - 3s \end{cases}$$

§11.6 Calculus of Vector-Valued Functions

7. Find the limit if it exists: $\lim_{t\to\pi/2}\langle\cos t, t^2+3, \tan t\rangle$

8. If $\mathbf{r}(t) = \langle\sqrt{t^2+1}, \cos t, e^{-3t}\rangle$, find $\mathbf{r}'(t)$.

9. Evaluate the definite integral: $\displaystyle\int_0^4 \left\langle 2te^{4t}, t^2-1, \frac{4t}{t^2+1}\right\rangle dt$

10. Find all values of t for which $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular. $\mathbf{r}(t) = \langle 2\cos t, \sin t \rangle$.

11. Find all values of t for which $\mathbf{r}'(t)$ is parallel to the xy-plane. $\mathbf{r}(t) = \langle \sqrt{t+1}, \cos t, t^4 - 8t^2 \rangle$.

12. Prove that if $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal for all t, then $|\mathbf{r}(t)| = \textit{constant}$.