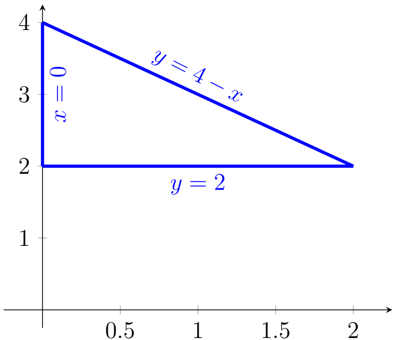
Recitation 08: Shell Method, Arc Length, & Physical Applications

Joseph Wells
Arizona State University

March 3, 2014

Example (13)**.** Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line. $y = 4 - x$, $y = 2$, $x = 0$, the line $y = 0$

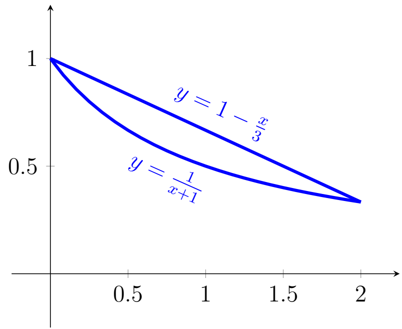
Since we are revolving around $y = 0$, we have to integrate with respect to y. So $y = 4 - x \Rightarrow x = 4 - y$ and our limits of integration become $y = 2$ and $y = 4 - (0) = 4$. Hence



$$
\begin{aligned}
V &= 2\pi \int_2^4 y(4 - y)\, dy \\
&= 2\pi \left(2y^2 - \frac{1}{3}y^3 \right) \Big|_2^4 \\
&= \frac{32\pi}{3}.
\end{aligned}
$$

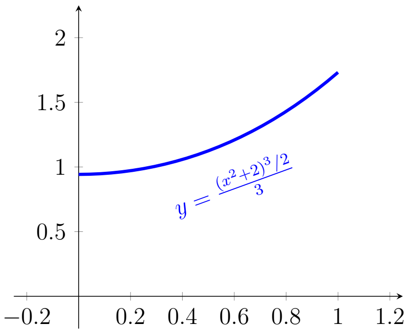
Example (14)**.** Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line. $y = \dfrac{1}{x+1}$, $y = 1 - \dfrac{x}{3}$, the line $y = 0$

Since we are revolving around $y = 0$, we have to integrate with respect to y. So $y = \frac{1}{x+1} \Rightarrow x = \frac{1}{y} - 1$ and $y = 1 - \frac{x}{3} \Rightarrow x = 3 - 3y$. Our limits of integration become the $y = \frac{1}{3}$ and $y = 1$. Hence



$$\begin{aligned} V &= 2\pi \int_{1/3}^{1} y(3 - 3y - (1/y - 1))\, dy \\ &= 2\pi \int_{1/3}^{1} (4y - 3y^2 - 1)\, dy \\ &= 2\pi (2y^2 - y^3 - y)\Big|_{1/3}^{1} = \frac{8\pi}{27}. \end{aligned}$$

Example (17)**.** Find the arc length of the following curves on the given interval by integrating with respect to x. $y = \dfrac{(x^2+2)^{3/2}}{3}$; $[0,1]$



$y' = x(x^2+2)^{1/2}$, so

$$1 + y'^2 = 1 + x^2(x^2+2) = x^4 + 2x^2 + 1 = (x^2+1)^2$$

Thus

$$L = \int_0^1 (x^2+1)\,dx = (x^3/3 + x)\Big|_0^1 = \frac{4}{3}$$

Example (21)**.** It takes 100 J of work to stretch a spring 0.5 m from its equilibrium position. How much work is needed to stretch it an additional 0.75 m?

Recall *Hooke's Law* for the force on a spring: $F(x) = kx$. To solve for the spring constant k, we use the following integral

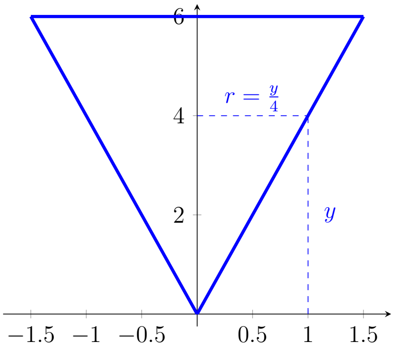
$$100 = \int_0^{0.5} kx \, dx = \frac{1}{8}k$$

So $k = 800$ and the amount of work required to move it an additional 0.75 m is

$$W = \int_{0.5}^{1.25} kx \, dx = (400x^2)\Big|_{0.5}^{1.25} = 525 \text{ J}.$$

Example (22)**.** A water tank is shaped like an inverted cone with height 6 m and a base radius 1.5 m. If the tank is full, how much work is required to pump the water to the level of the top of the tank and out of the tank?

Let the vertex of the cone be at $(0, 0)$. By similar triangles, a horizontal slice at height y is has area $\pi y^2/16$, and it must move $6 - y$ meters to get to the top. So, letting $\rho = 1\,\text{kg/m}^3$ (the density of water),



$$\begin{aligned}
W &= \int_0^6 \rho g \pi \frac{y^2}{16}(6 - y)\, dy \\
&= (\pi \rho g/16) \int_0^6 (6y^2 - y^3)\, dy \\
&= (\pi \rho g/16)(2y^3 - y^4/4)\Big|_0^6 \\
&= (\pi \rho g/16)(108) = 66,150\pi \text{ J}.
\end{aligned}$$

Assignment

Recitation Notebook:
§6.1 - #1
§6.4 - #2, #4
§6.5 - #1, #3
§6.6 - #2

As always, you may work in groups, but every member must individually submit a homework assignment.