Recitation 07: Area Between Curves & Volumes of Revolution, Pt I

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Revolutions about the x-axis:

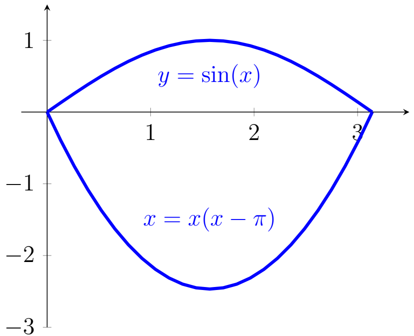
Disk Method	$V = \int_a^b \pi f(x)^2 \, dx$
Washer Method	$V = \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] \, dx$

Revolutions about the y-axis:

Disk Method	$V = \int_a^b \pi f(y)^2 \, dy$
Washer Method	$V = \int_a^b \pi \left[f(y)^2 - g(y)^2 \right] \, dy$

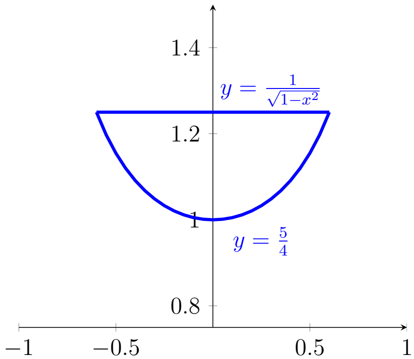
Example (Rec Notebk: §6.2, # 2)**. Region between curves** Sketch the region and find its area. The region bounded by $y = \sin(x)$ and $y = x(x - \pi)$ for $0 \le x \le \pi$.

$$\begin{aligned} A &= \int_0^{\pi} [\sin(x) - x(x-\pi)]\ dx \\ &= \int_0^{\pi} \left[\sin(x) - x^2 + \pi x)\right]\ dx \\ &= \left[-\cos(x) - \frac{x^3}{3} + \frac{\pi x^2}{2}\right]_0^{\pi} \\ &= 2 + \frac{\pi^3}{6} \end{aligned}$$



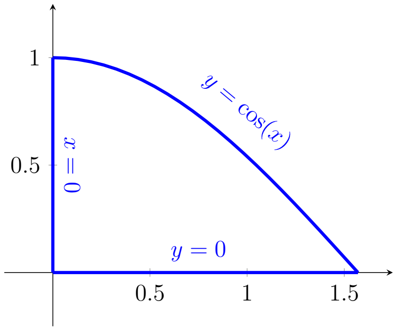
Example (Rec Notebk: §6.2, # 3)**. Region between curves** Sketch the region and find its area. The region bounded by $y = 5/4$ and $y = \frac{1}{\sqrt{1-x^2}}$.

$$\begin{aligned}
A &= \int_{-3/5}^{3/5} \left[\frac{5}{4} - \frac{1}{\sqrt{1-x^2}}\right]\, dx \\
&= 2\int_{0}^{3/5} \left[\frac{5}{4} - \frac{1}{\sqrt{1-x^2}}\right]\, dx \\
&= 2\left[\frac{5x}{4} - \arcsin(x)\right]_0^{3/5} \\
&= \frac{3}{2} - 2\arcsin\left(\frac{3}{5}\right).
\end{aligned}$$



Example (Rec Notebk: §6.3, # 2)**. Disk method** Let R be the region bounded by the following curves. Use the disk method to find the volume of the solid generated when R is revolved around the x-axis. $y = \cos(x)$, $y = 0$, $x = 0$ (Recall that $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$.)

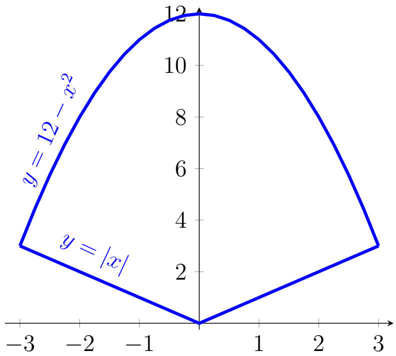
$$\begin{aligned}
V &= \int_0^{\pi/2} \pi \cos^2(x)\,dx \\
&= \int_0^{\pi/2} \frac{\pi(1+\cos(2x))}{2}\,dx \\
&= \left[\frac{\pi x}{2} + \frac{\pi \sin(2x)}{4}\right]_0^{\pi/2} \\
&= \frac{\pi^2}{4}.
\end{aligned}$$



Example (Rec Notebk: §6.3, # 4). **Washer method** Let R be the region bounded by the following curves. Use the washer method to find the volume of the solid generated when R is revolved around the x-axis. $y = |x|$, $y = 12 - x^2$.

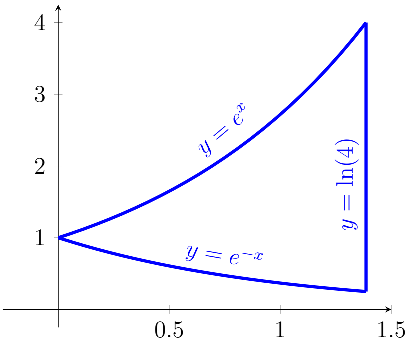
Notice the area is symmetric about the y-axis, so we can consider only the right half and double it.

$$
\begin{aligned}
V &= \int_{-3}^{3} \pi \left[(12 - x^2)^2 - x^2\right]\, dx \\
&= 2\pi \int_{0}^{3} \left[(12 - x^2)^2 - x^2\right]\, dx \\
&= 2\pi \int_{0}^{3} \left[x^4 - 25x^2 + 144\right]\, dx \\
&= 2\pi \left[\frac{x^5}{5} - \frac{25x^3}{3} + 144x\right]_0^3 \\
&= \frac{2556\pi}{5}.
\end{aligned}
$$



Example (Rec Notebk: §6.3, # 6). **Solids of Revolution** Find the volume of the solid of revolution. Sketch the region in question. The region bounded by $y = e^{-x}$, $y = e^{x}$, $x = 0$, and $x = \ln(4)$ revolved about the x-axis.

$$
\begin{aligned}
V &= \int_0^{\ln(4)} \pi\left[(e^x)^2 - (e^{-x})^2\right]\,dx \\
&= \int_0^{\ln(4)} \pi\left[e^{2x} - e^{-2x}\right]\,dx \\
&= \int_0^{\ln(4)} \pi\left[e^{2x} - e^{-2x}\right]\,dx \\
&= \left[\frac{\pi e^{2x}}{2} - \frac{\pi e^{-2x}}{2}\right]_0^{\ln(4)} \\
&= \frac{225\pi}{32}.
\end{aligned}
$$



Assignment

Recitation Notebook:
§6.2 - #1, #5
§6.3 - #1, #3, #5

As always, you may work in groups, but every member must individually submit a homework assignment.