MAT271 Exam 4 Review

Spring 2014

This is not a complete list of topics covered in class, but merely a compilation of supplemental exercises from each section. You should still review class notes, previous supplemental exercises, and the practice exam posted on ASU's MAT271 course page:
`http://math.asu.edu/first-year-math/mat-271-calculus-analytic-geometry-ii`

§10.1

Find the arc length of the following parametric equations on the given interval.

1. $x = t^2$, $y = 2t$; $[0, 2]$

2. $x = 5t + 1$, $y = 12t + 98778516$; $[-1, 0]$

3. $x = e^{-t}\cos(t)$, $y = e^{-t}\sin(t)$; $\left[0, \frac{\pi}{2}\right]$

4. $x = \sqrt{t}$, $y = 3t - 1$; $[0, 1]$.

§10.2

Convert the following equations to Cartesian coordinates. Describe the resulting curve.

5. $r = \cot(\theta)\csc(\theta)$

6. $1 = 2r\cos(\theta) + 3r\sin(\theta)$

7. $r = \sin(\theta)\sec^2(\theta)$

8. $r = 8\sin(\theta)$

§10.3

Find the slope of the tangent line to the polar curve at the given points. At the points where the curve intersects the origin (when this occurs), find the equation of the tangent line in polar coordinates.

9. $r = 4 + \sin(\theta)$; $\left(3, \frac{3\pi}{2}\right)$

10. $r = 2\theta$; $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

11. Find the area of the region inside one leaf of the rose $r = 3\sin(2\theta)$.

12. Find the area of the region inside the lemniscate $r^2 = 2\sin(2\theta)$ and outside the circle $r = 1$.

§10.4

13. Find an equation of the parabola that opens downward with directrix $y = 6$ and vertex at $(0, 0)$.

14. Find an equation of the hyperbola with vertices $(0, \pm 4)$, asymptotes $y = \pm 2x$, and center at $(0, 0)$.

15. Find an equation of an ellipse with vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$, and center at $(0, 0)$.

16. Find an equation of the line tangent to $y^2 - \dfrac{x^2}{64} = 1$ at the point $\left(6, -\frac{5}{4}\right)$.

Solutions

Although these were all taken from the textbook's answer key, there may still be typos.

1. $2\sqrt{5} + \ln(2 + \sqrt{5}) \approx 5.916$

2. 13

3. $\sqrt{2}\left(1 - e^{-\pi/2}\right) \approx 1.12$

4. $\frac{1}{12}\left[\ln(\sqrt{37} + 6) + 6\sqrt{37}\right] \approx 3.249$

5. $x = y^2$; horizontal parabola

6. $y = -\frac{2}{3}x + \frac{1}{3}$; line

7. $y = x^2$; vertical parabola

8. $x^2 + (y - 4)^2 = 16$; circle

9. $\dfrac{dy}{dx} = \dfrac{\cos(\theta)\sin(\theta) + (4 + \sin(\theta))\cos(\theta)}{\cos^2(\theta) - (4 + \sin(\theta))\sin(\theta)}$
At $(4, 0)$, $\frac{dy}{dx} = 4$. At $\left(3, \frac{3\pi}{2}\right)$, $\frac{dy}{dx} = 0$.

10. $\dfrac{dy}{dx} = \dfrac{2\sin(\theta) + 2\theta\cos(\theta)}{2\cos(\theta) - 2\theta\sin(\theta)}$
At $\left(\frac{\pi}{2}, \pi 4\right)$, $\frac{dy}{dx} = \frac{\sqrt{2} + \frac{\pi}{4}\sqrt{2}}{\sqrt{2} - \frac{\pi}{4}\sqrt{2}} \approx 8.32$.

11. $A = 2 \cdot \dfrac{1}{2}\displaystyle\int_0^{\pi/10} \cos^2(5\theta)\, d\theta = \dfrac{\pi}{40}$

12. $A = 2 \cdot \dfrac{1}{2}\displaystyle\int_{\pi/12}^{5\pi/12} (2\sin(2\theta) - 1)\, d\theta = \sqrt{3} - \dfrac{\pi}{3}$

13. $x^2 = -24y$

14. $\frac{y^2}{4} - x^2 = 1$

15. $\frac{x^2}{36} + \frac{y^2}{20} = 1$

16. $y = -\frac{3}{40}x - \frac{4}{5}$