MAT271 Exam 1 Review

Spring 2014

This is not a complete list of topics covered in class, but merely a compilation of supplemental exercises from each section. You should still review class notes and the practice exam posted on ASU's MAT271 course page:
`http://math.asu.edu/first-year-math/mat-271-calculus-analytic-geometry-ii`

§5.5 u-Substitutions

1. $\displaystyle\int \frac{x^2}{16-x^6}\,dx$

2. $\displaystyle\int (x+1)^9\,dx$

3. $\displaystyle\int_3^5 2x\sqrt{x^2-9}\,dx$

4. $\displaystyle\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

§7.1 Integration by Parts

5. $\displaystyle\int e^t\sin(t)\,dt$

6. $\displaystyle\int x\arcsin(2x)\,dx$

7. $\displaystyle\int \ln(\sqrt{x^2-1})\,dx$

8. $\displaystyle\int x\sqrt{x-5}\,dx$

§7.2 Trigonometric Integrals

9. Write the three pythagorean identities for trigonometric functions.

10. Write down the double angle formulas for $\sin(2x)$ and $\cos(2x)$.

11. Write down the power-reducing formulas for $\sin^2(x)$ and $\cos^2(x)$.

12. $\displaystyle\int \cos^3(\pi x-1)\,dx$

13. $\displaystyle\int \sec^4\left(\frac{x}{2}\right)\,dx$

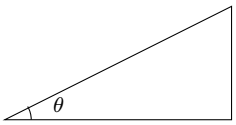
14. $\displaystyle\int \frac{1}{1-\sin(\theta)}\,d\theta$

15. $\displaystyle\int \tan(\theta)\sec^4(\theta)\,d\theta$

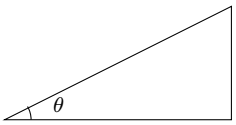
§7.3 Trigonometric Substitutions

Complete the reference triangle for the given trigonometric substitution.

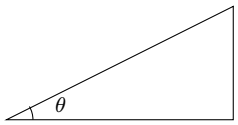
16. $x = a\sin(\theta)$



17. $x = a\sec(\theta)$



18. $x = a\tan(\theta)$



19. $\displaystyle\int \frac{-12}{x^2\sqrt{4-x^2}}\,dx$

20. $\displaystyle\int \frac{x^3}{\sqrt{4+x}}\,dx$

21. $\displaystyle\int \sqrt{9-4x^2}\,dx$

22. $\displaystyle\int_0^{\pi/2} \frac{\sin(\theta)}{1+2\cos^2(\theta)}\,d\theta$

§7.4 Partial Fractions

23. From the following equation, write down the augmented coefficient matrix: $8x^4 + 2x^2 + 3x - 9 = (A+B)x^4 + Cx^3 + (D-2E-A-3B+C)x^2 + (E-12D)x + (A+B+C+D-E)$

24. Using a calculator, put the previous matrix into *reduced row echelon form* (RREF).

24. $\displaystyle\int \frac{x^2-1}{x^3+x}\,dx$

25. $\displaystyle\int \frac{x^2+x+3}{x^4+6x^2+9}\,dx$

26. $\displaystyle\int_1^5 \frac{x-1}{x^2(x+1)}$

27. $\displaystyle\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}\,dx$

§7.5 Other Integration Strategies

28. $\displaystyle\int \frac{x}{1+e^{-x^2}}\,dx$

29. $\displaystyle\int x^3\sin(x)\,dx$

30. $\displaystyle\int \frac{\sqrt{3-5x}}{2x}\,dx$

31. $\displaystyle\int x\sqrt{x^4-9}\,dx$

§7.7 Improper Integrals

32. $\displaystyle\int_1^\infty \frac{dx}{x}$

33. $\displaystyle\int_0^\infty \frac{1}{x^2+1}\,dx$

34. $\displaystyle\int_{-\infty}^\infty \frac{e^x}{1+e^{2x}}\,dx$

35. $\displaystyle\int_{-1}^1 \frac{1}{x^2}\,dx$

Solutions

1. Let $u = \frac{x^3}{4}$ and $du = \frac{3}{4}x^2\,dx$. Then

$$\begin{aligned}\int \frac{x^2}{16-x^6}\,dx &= \frac{4}{3}\int \frac{x^2}{16-16u^2}\,dx \\ &= \frac{1}{12}\int \frac{x^2}{1-u^2}\,du \\ &= \frac{1}{12}\arcsin(u) + C \\ &= \frac{1}{12}\arcsin\left(\frac{x^3}{4}\right) + C.\end{aligned}$$

2. Let $u = x+1$ and $du = dx$. Then

$$\begin{aligned}\int (x+1)^9\,dx &= \int u^9\,du \\ &= \frac{1}{10}u^{10} + C \\ &= \frac{1}{10}(x+1)^{10} + C.\end{aligned}$$

3. Let $u = x^2 - 9$ and $du = 2x\,dx$. Then

$$\begin{aligned}\int_{x=3}^{x=5} 2x\sqrt{x^2-9}\,dx &= \int_{u=0}^{u=16} \sqrt{u}\,du \\ &= \frac{2}{3}u^{2/3}\Big|_{u=0}^{u=16} \\ &= \frac{128}{3}.\end{aligned}$$

4. Let $u = e^x + e^{-x}$ and $du = (e^x - e^{-x})\,dx$. Then

$$\begin{aligned}\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx &= \int \frac{1}{u}\,du \\ &= \ln(u) + C \\ &= \ln\left(e^x + e^{-x}\right) + C.\end{aligned}$$

5. Let $u = \sin(t)$ and $dv = e^t\,dt$. Then $du = \cos(t)\,dt$ and $v = e^t$, so

$$\int e^t \sin(t)\,dt = e^t\sin(t) - \int e^t\cos(t)\,dt.$$

Now let $u' = \cos(t)$ and $dv' = e^t\,dt$. Then $du' = -\sin(t)\,dt$ and $v' = e^t$, so

$$\begin{aligned}\int e^t\sin(t)\,dt &= e^t\sin(t) - e^t\cos(t) - \int e^t\sin(t)\,dt \\ 2\int e^t\sin(t)\,dt &= e^t\sin(t) - e^t\cos(t) + C \\ \int e^t\sin(t)\,dt &= \frac{1}{2}e^t\left[\sin(t) - \cos(t)\right] + C.\end{aligned}$$

6. Let $u = x$ and $dv = \arctan(2x)\,dx$. Then $du = dx$ and $v = \frac{1}{1+(2x)^2}$, so

7. Let $u = \ln(\sqrt{x^2-1})$ and $dv = dx$. Then $du = \frac{1}{x^2-1}\,dx$ and $v = x$, so

8. Let $u = x$

23.

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -3 & 1 & 1 & -2 & 2 \\ 0 & 0 & 0 & -12 & 1 & 3 \\ 1 & 1 & 1 & -1 & 0 & -9 \end{array}\right)$$

24.

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{423}{2} \\ 0 & 1 & 0 & 0 & 0 & -\frac{407}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 17 \\ 0 & 0 & 0 & 0 & 1 & 207 \end{array}\right)$$