Recitation 08: Physical Applications & Intro to Sequences

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Example (21)**.** It takes 100 J of work to stretch a spring 0.5 m from its equilibrium position. How much work is needed to stretch it an additional 0.75 m?

Recall *Hooke's Law* for the force on a spring: $F(x) = kx$. To solve for the spring constant k, we use the following integral

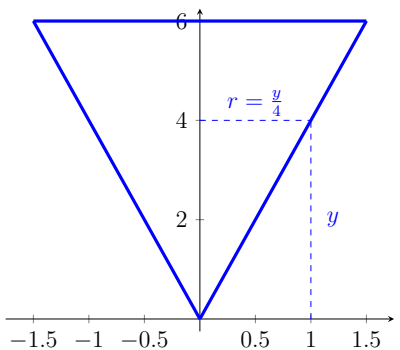
$$100 = \int_0^{0.5} kx\,dx = \frac{1}{8}k$$

So $k = 800$ and the amount of work required to move it an additional 0.75 m is

$$W = \int_{0.5}^{1.25} kx\,dx = (400x^2)\Big|_{0.5}^{1.25} = 525\,\text{J}.$$

Example (22)**.** A water tank is shaped like an inverted cone with height 6 m and a base radius 1.5 m. If the tank is full, how much work is required to pump the water to the level of the top of the tank and out of the tank?

Let the vertex of the cone be at $(0, 0)$. By similar triangles, a horizontal slice at height y is has area $\pi y^2/16$, and it must move $6 - y$ meters to get to the top. So, letting $\rho = 1\,\text{kg/m}^3$ (the density of water),



$$\begin{aligned} W &= \int_0^6 \rho g \pi \frac{y^2}{16}(6 - y)\,dy \\ &= (\pi\rho g/16)\int_0^6 (6y^2 - y^3)\,dy \\ &= (\pi\rho g/16)(2y^3 - y^4/4)\Big|_0^6 \\ &= (\pi\rho g/16)(108) = 66,150\pi\,\text{J}. \end{aligned}$$

Example (Rec Notebk: §8.1, #5)**. Limits from Graphs** Consider the following sequence: $a_n = \frac{n^2}{n^2-1}$; $n = 2, 3, 4$.

a. Find the first four terms of the sequence.

b. Based on part (a) and the figure, determine a plausible limit of the sequence.

Solution.

a. $\frac{4}{3}, \frac{9}{8}, \frac{16}{15}, \frac{25}{24}$

b. It looks like it is approaching 1, and in fact, it is.

Example (Rec Notebk: §8.1, #6)**. Repeating decimals**

a. Write the following repeating decimal as an infinite series: $0.0\bar{2}7 = 0.027027027\ldots$.

b. Find the limit of the sequence of partial sums for the infinite series and express it as a fraction.

Solution.

a. $0.0\bar{2}7 = \sum_{k=1}^{\infty} 27(0.001)^k$.

b. We can rewrite the series as

$$\begin{aligned}\sum_{k=1}^{\infty} 27(0.001)^k &= -27 + \sum_{k=0}^{\infty} 27(0.001)^k \\ &= -27 + \sum_{k=0}^{\infty} 27\left(\frac{1}{1000}\right)^k \\ &= -27 + \frac{27}{1-\frac{1}{1000}} = \frac{27}{999} = \frac{1}{37}\end{aligned}$$

Assignment

Recitation Notebook:
§6.6 - #2, #4, #5
§8.1 - #1, #2

As always, you may work in groups, but every member must individually submit a homework assignment.