Recitation 07: Shell Method & Arc Length

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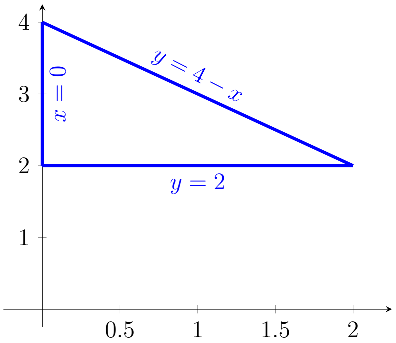
Example (13)**.** Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line. $y = 4 - x$, $y = 2$, $x = 0$, the line $y = 0$

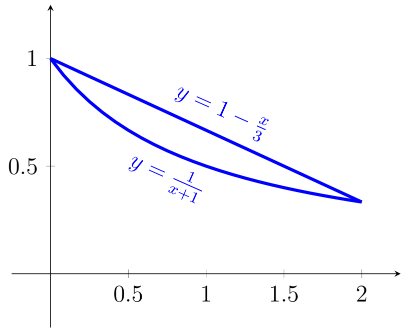
Since we are revolving around $y = 0$, we have to integrate with respect to y. So $y = 4 - x \Rightarrow x = 4 - y$ and our limits of integration become $y = 2$ and $y = 4 - (0) = 4$. Hence



$$
\begin{aligned}
V &= 2\pi \int_2^4 y(4-y)\,dy \\
&= 2\pi \left(2y^2 - \frac{1}{3}y^3\right)\Bigg|_2^4 \\
&= \frac{32\pi}{3}.
\end{aligned}
$$

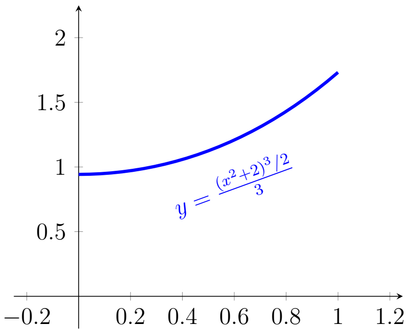
Example (14)**.** Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line. $y = \dfrac{1}{x+1}$, $y = 1 - \dfrac{x}{3}$, the line $y = 0$

Since we are revolving around $y = 0$, we have to integrate with respect to y. So $y = \frac{1}{x+1} \Rightarrow x = \frac{1}{y} - 1$ and $y = 1 - \frac{x}{3} \Rightarrow x = 3 - 3y$. Our limits of integration become the $y = \frac{1}{3}$ and $y = 1$. Hence



$$\begin{aligned} V &= 2\pi \int_{1/3}^{1} y(3 - 3y - (1/y - 1))\, dy \\ &= 2\pi \int_{1/3}^{1} (4y - 3y^2 - 1)\, dy \\ &= 2\pi(2y^2 - y^3 - y)\Big|_{1/3}^{1} = \frac{8\pi}{27}. \end{aligned}$$

Example (17)**.** Find the arc length of the following curves on the given interval by integrating with respect to x. $y = \dfrac{(x^2+2)^{3/2}}{3}$; $[0, 1]$



$y' = x(x^2+2)^{1/2}$, so

$$1 + y'^2 = 1 + x^2(x^2+2) = x^4 + 2x^2 + 1 = (x^2+1)^2$$

Thus

$$L = \int_0^1 (x^2+1)\,dx = (x^3/3 + x)\Big|_0^1 = \frac{4}{3}$$

Assignment

Recitation Notebook:
§6.4 - #2, #4
§6.5 - #1, #3
and the following worksheet:
`http://math.joedub.net/teaching/fall2014/homework07.pdf`

As always, you may work in groups, but every member must individually submit a homework assignment.