Recitation 05: Alternative Integration Methods & Application of Integration

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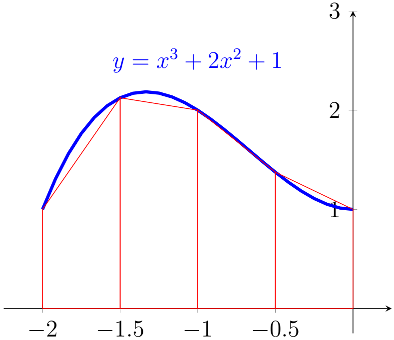
September 19, 2014

Sometimes we may have to approximate an integral numerically. We have already seen Riemann sums, but there are also two other common methods of doing it.

Trapezoid Rule



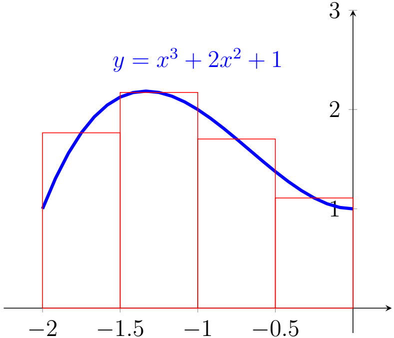
$$\int_a^b f(x)\,dx \approx T(n)$$

where

$$T(n) = \left(\frac{f(x_0)}{2} + \sum_{k=1}^{n-1} f(x_k) + \frac{f(x_n)}{2} \right) \Delta x,$$

n is the number of intervals, $\Delta x = \frac{(b-a)}{n}$, and $x_k = a + k\Delta x$.

Midpoint Rule



$$\int_a^b f(x)\,dx \approx M(n)$$

where

$$M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1} + x_k}{2}\right)\Delta x,$$

n is the number of intervals, $\Delta x = \frac{(b-a)}{n}$, $x_k = a + k\Delta x$.

For the given example where $f(x) = x^3 + 2x^2 + 1$ on $[-2, 0]$, let's calculate $M(n)$ and $T(n)$ for $n = 4$ and compare it to the definite integral $\int_{-2}^{0} f(x)\, dx$.

[insert work here]

Example. Let's say we're interested in blasting a 100 kg ($\approx$ 220 lbs) classmate off into space. How much work has to be done to escape Earth's gravity?

Solution.

Following from the universal law of gravitation, the amount of work required to escape Earth's gravity is given by

$$\int_{r_0}^{\infty} \frac{Gm_1m_2}{r^2}\,dr,$$

where

$$\begin{aligned}
r_e &= 6.38 \times 10^6\,\text{m (the approximate radius of Earth)}\\
G &= 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2 \text{ (the Universal Gravitation Constant)}\\
m_1 &= 100\,\text{kg (the mass of your classmate)}\\
m_2 &= 6 \times 10^{24}\,\text{kg (the mass of Earth).}
\end{aligned}$$

So since this integral is improper, we use limits to see that

$$\begin{aligned}
\int_{r_0}^{\infty} \frac{Gm_1m_2}{r^2}\,dr &= Gm_1m_2 \int_{r_0}^{\infty} \frac{1}{r^2}\,dr \\
&= Gm_1m_2 \lim_{t\to\infty} \int_{r_0}^{t} \frac{dr}{r^2} \\
&= Gm_1m_2 \lim_{t\to\infty} \left[-\frac{1}{r}\right]_{r_0}^{t} \\
&= Gm_1m_2 \lim_{t\to\infty} \left[\frac{1}{r_0} - \frac{1}{t}\right] \\
&= \frac{Gm_1m_2}{r_0} \\
&\approx 6.292 \times 10^9\,\mathrm{N}\cdot\mathrm{m} = 6.292 \times 10^9\,\mathrm{J}.
\end{aligned}$$

Assignment

Recitation Notebook:
§7.5 - #3, #4, (state integral number in Table of Integrals)
§7.6 - #1, #2,
and the following worksheet:
`http://math.joedub.net/teaching/mat271_fall2014/homework05.pdf`

As always, you may work in groups, but every member must individually submit a homework assignment.