Recitation 03: Trig Substitutions & Partial Fractions

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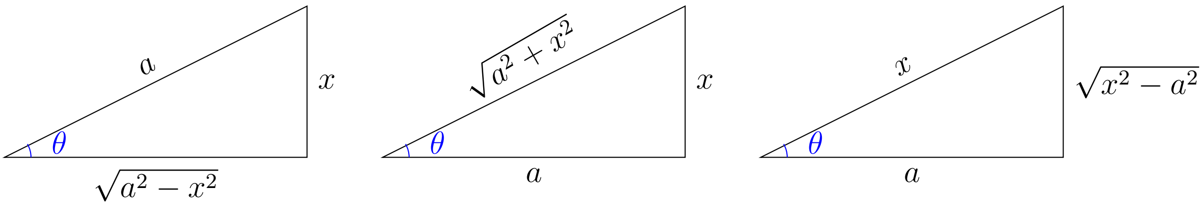
When we see a polynomial under a square root, often times it is easiest to approach with a trig substitution. Your book builds it up more intuitively, but the results for the three basic trig substitutions are as follows:

Integrand	Trig Substitution
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$.

Each of these corresponds to a reference triangle



Example (Rec Notebk: §7.3, # 1)**.** Evaluate the following integral: $\displaystyle\int \frac{dx}{\sqrt{16+4x^2}}$

Solution.

Here $a = 2$, so we use the trig substitution $x = 2\tan(\theta)$ and $dx = 2\sec^2(\theta)\,d\theta$:

$$\begin{aligned}
\int \frac{dx}{\sqrt{16+4x^2}} &= \int \frac{dx}{2\sqrt{4+x^2}} \\
&= \int \frac{2\sec^2(\theta)d\theta}{2\sqrt{4(1+\tan^2(\theta)}} \\
&= \frac{1}{2}\int \frac{\sec^2(\theta)d\theta}{\sec(\theta)} \\
&= \frac{1}{2}\int \sec(\theta)\,d\theta \\
&= \frac{1}{2}\ln|\sec(\theta)+\tan(\theta)|+C
\end{aligned}$$

Using our reference triangle, we get that $\sec(\theta) = \frac{\sqrt{4+x^2}}{2}$ and $\tan(\theta) = \frac{x}{2}$, so

$$\frac{1}{2} \ln |\sec(\theta) + \tan(\theta)| + C = \frac{1}{2} \ln \left| \frac{1}{2}(\sqrt{4 + x^2} + x) \right| + C.$$

Example. Find the partial fraction decomposition for $\dfrac{6}{x^2-1}$

Solution.

$$\frac{6}{x^2-1} = \frac{6}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}.$$

We then have that

$$\begin{aligned} 6 = A(x-1) + B(x+1) \qquad & \implies 0 = (A+B)x \\ & \implies 6 = B - A. \end{aligned}$$

Solving, we get that $A = -3$ and $B = 3$. Hence

$$\frac{6}{x^2-1} = \frac{-3}{x+1} + \frac{3}{x-1}.$$

Assignment

Recitation Notebook:
§7.3 - #2, #3, #4
§7.4 - #1, #2
And the following worksheet:
`http://math.joedub.net/teaching/mat271_fall2014/homework03.pdf`

As always, you may work in groups, but every member must individually submit a homework assignment.