MAT271 Exam 3 Review

Spring 2014

This is not a complete list of topics covered in class, but merely a compilation of supplemental exercises from each section. You should still review class notes and the practice exam posted on ASU's MAT271 course page:
`http://math.asu.edu/first-year-math/mat-271-calculus-analytic-geometry-ii`

§8.1 An Overview (of Sequences)

For each fo the following sequences,
a. Find the next two terms in the sequence.
b. Find a recurrence relation that generates the sequence (supply the initial value of the index and the first term of the sequence).
c. Find an explicit formula for the general n^{th} term of the sequence.

1. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\right\}$
2. $\{1, -2, 3, -4, 5, \ldots\}$
3. $\{1, 2, 4, 8, 16, \ldots\}$
4. $\{1, 3, 9, 27, 81, \ldots\}$

§8.2 Sequences

Find the limit of the following sequences, or state that the limit does not exist. If the sequence is geometric, determine whether or not the sequence is monotone.

5. $\left\{\frac{2e^{n+1}}{e^n}\right\}$
6. $\left\{\left(\frac{1}{n}\right)^{1/n}\right\}$
7. $\{(-0.003)^n\}$
8. $\left\{\frac{n\sin^3(n)}{n+1}\right\}$

§8.3 Infinite Series

For each of the following sums/series, determine the type of sum/series. Then evaluate the sum/series, or state that it diverges.

9. $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \cdots + \frac{1}{2916}$
10. $\displaystyle\sum_{m=2}^{\infty} \frac{5}{2^m}$
11. $\displaystyle\sum_{k=1}^{\infty} 3\left(\frac{-1}{8}\right)^{3k}$
12. $\displaystyle\sum_{k=1}^{\infty} \left[\tan^{-1}(k+1) - \tan^{-1}(k)\right]$

§8.4 The Divergence and Integral Tests

Determine whether or not the following series converge or diverge, and state the test used.

13. $\displaystyle\sum_{k=1}^{\infty}\sqrt{\frac{k+1}{k}}$

14. $\displaystyle\sum_{k=1}^{\infty}\frac{1}{(3x+1)(3x+4)}$

15. $\displaystyle\sum_{k=0}^{\infty}\frac{10}{k^2+9}$

16. $\displaystyle\sum_{k=2}^{\infty}\frac{4}{k\ln^2(k)}$

§8.5 The Ratio, Root, and Comparison Tests

Determine whether or not the following series converge or diverge, and state the test used.

17. $\displaystyle\sum_{k=1}^{\infty}\frac{(k!)^3}{(3k)!}$

18. $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k^2\ln(k)}$

19. $\displaystyle\sum_{k=1}^{\infty}\sin^2\left(\frac{1}{k}\right)$

20. $\frac{1}{1!}+\frac{4}{2!}+\frac{9}{3!}+\frac{16}{4!}+\cdots$

§8.6 Alternating Series

Determine whether or not the following series converge or diverge.

21. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^{10}+2k^5+1}{k(k^{10}+1)}$

22. $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{\sqrt{k^2+4}}$

23. $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k k}{2k+1}$

24. $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k\tan^{-1}(k)}{k^3}$

§9.1 Approximating Functions with Polynomials

Find the n^{th}-order Taylor polynomial for the given functions centered at the given point a, where $n = 0, 1, 2$.

25. $f(x) = \ln(1-x)$, $a = 0$

26. $f(x) = \tan(x)$, $a = 0$

27. $f(x) = \sin(x)$, $a = \frac{\pi}{4}$

28. $f(x) = \sqrt{x}$, $a = 9$

§9.2 Properties of Power Series

Find the interval and radius of convergence for each of the following power series.

29. $\sum \frac{k^2 x^{2k}}{k!}$

30. $\sum \frac{(x-1)^k k^k}{(k+1)^k}$

Find the power series representation for g centered at 0 by differentiating or integrating the power series for f (perhaps more than once). Give the interval of convergence for the resulting series.

31. $g(x) = \frac{1}{(x-4)^4}$ using $f(x) = \frac{1}{1-x}$

32. $g(x) = \ln(1-3x)$ using $f(x) = \frac{1}{1-3x}$

§9.3 Taylor Series

For the following series...
a. Find the first four nonzero terms of the Taylor series centered at a.
b. Write the power series using summation notation.

33. $f(x) = (1+x^2)^{-1}$, $a = 0$

34. $f(x) = \tan^{-1}(x)$, $a = 0$

35. $f(x) = \sin(x)$, $a = \frac{\pi}{2}$

36. $f(x) = \ln(x)$, $a = 3$

§9.4 Working with Taylor Series

Evaluate the following limits using Taylor series.

37. $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

38. $\lim_{x\to 0} \frac{3\tan(x) - 3x - x^3}{x^5}$

39. $\lim_{x\to 0} \frac{\sin(x) - \tan(x)}{3x^3\cos(x)}$

40. $\lim_{x\to 2} \frac{x-2}{\ln(x-1)}$

Solutions

Although these were all taken from the textbook's answer key, there may still be typos.

1.
a. $\frac{1}{32}$, $\frac{1}{64}$
b. $a_1 = 1$, $a_n + 1 = \frac{a_n}{2}$
c. $a_n = \frac{1}{2^{n-1}}$

2.
a. -6, 7
b. $a_1 = 1$, $a_{n+1} = (-1)^n(|a_n| + 1)$
c. $a_n = (-1)^{n+1}n$

3.
a. 32, 64
b. $a_1 = 1$, $a_{n+1} = 2a_n$
c. $a_n = 2^{n-1}$

4.
a. 243, 729
b. $a_1 = 1$, $a_{n+1} = 3a_n$
c. $a_n = 3^{n-1}$

5. $2e$

6. 1

7. 0, not monotone

8. diverges

9. $\frac{1093}{2916}$

10. $\frac{5}{2}$

11. $\frac{-1}{171}$

12. $\frac{\pi}{4}$

13. Divergent by Divergence Test.

14. Converges by Integral Test.

15. Converges by Integral Test.

16. Converges by Integral Test.

17. Converges by Ratio Test.

18. Converges by Comparison Test ($\frac{1}{k^{\ln(k)}} < \frac{1}{k^2}$ for $k > 10$).

19. Converges by Comparison Test ($\frac{1}{k^2}$).

20. Converges by Ratio Test.

21. Converges.

22. Converges.

23. Diverges by Divergence Test.

24. Converges by Comparison Test ($\frac{\pi}{2}\frac{1}{k^3}$). Converges absolutely.

25. $p_0(x) = 0$
$p_1(x) = -x$
$p_2(x) = -x - \frac{1}{2}x^2$

26. $p_0(x) = 0$
$p_1(x) = x$
$p_2(x) = x$

27. $p_0(x) = \frac{\sqrt{2}}{2}$
$p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$
$p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$

28. $p_0(x) = 10$
$p_1(x) = 10 + \frac{(x-100)}{20}$
$p_2(x) = 10 + \frac{(x-100)}{20} - \frac{(x-100)^2}{8000}$

29. Radius: ∞
Interval: $(-\infty, \infty)$

30. Radius: 1
Interval: $(0, 2)$

31. $g(x) = \frac{1}{6}\sum_{k=0}^{\infty}(k+1)(k+2)(k+3)x^k$
Interval: $(-1, 1)$

32. $g(x) = -\sum_{k=1}^{\infty}\frac{3^k}{k}x^k$
Interval: $\left[-\frac{1}{3}, \frac{1}{3}\right)$

33.
a. $1 - x^2 + x^4 - x^6 + \cdots$
b. $\sum_{k=0}^{\infty}(-1)^k x^{2k}$

34.
a. $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$
b. $\sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1}x^{2k+1}$

35.
a. $1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2} + \frac{\left(x-\frac{\pi}{2}\right)^5}{24} - \frac{\left(x-\frac{\pi}{2}\right)^6}{720} + \cdots$
b. $\sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k)!}\left(x - \frac{\pi}{2}\right)^{2k}$

36.
a. $\ln(4) + \frac{(x-3)}{3} - \frac{(x-3)^2}{18} + \frac{(x-3)^3}{81} + \cdots$
b. $\ln(3) + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k3^k}(x-3)^k$.

37. 2

38. $\frac{2}{5}$

39. $-\frac{1}{6}$

40. 1