Section 3.4

1. THEOREM 3.11

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

$$\lim_{x \to -3} \frac{\sin(x+3)}{x^2 + 8x + 15} = \lim_{x \to -3} \frac{\sin(x+3)}{(x+3)(x+5)}$$

LET $y = x + 3$

THEN

$$= \lim_{y \to 0} \frac{\sin(y)}{y(y+2)}$$

$$= \left[\lim_{y \to 0} \frac{\sin(y)}{y}\right]\left[\lim_{y \to 0} \frac{1}{y+2}\right]$$

APPLY THEOREM 3.11

$$= 1 \cdot \tfrac{1}{2} = \boxed{\tfrac{1}{2}.}$$

2. $$\frac{d}{dx}\left[\tan(x)\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$$

$$= \frac{[\sin(x)]'\cos(x) - \cos(x)[\sin(x)]'}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)}$$

$$= \boxed{\sec^2(x)}$$

4. $y = \dfrac{\sin(x)}{1+\cos(x)}$ QUOTIENT RULE.

$$y' = \frac{\cos(x)(1+\cos(x)) - (-\sin(x))\sin(x)}{(1+\cos(x))^2}$$

$$= \frac{\cos(x) + \cos^2(x) + \sin^2(x)}{(1+\cos(x))^2}$$

$$= \frac{\cos(x) + 1}{(1+\cos(x))^2}$$

$$= \boxed{\frac{1}{1+\cos(x)}}.$$

Section 3.6

1.

$$\frac{d}{dx}\left[\left[(x+2)(3x^3+3x)\right]^4\right]$$

$$= 4\left[(x+2)(3x^3+3x)\right]^3\left[(x+2)(3x^3+3x)\right]'$$

$$= 4\left[(x+2)(3x^3+3x)\right]^3\left[(x+2)'(3x^3+3x) + (x+2)(3x^3+3x)'\right]$$

$$= \boxed{4\left[(x+2)(3x^3+3x)\right]^3\left[(3x^3+3x) + (x+2)(9x^2+3)\right]}$$

2.

$$h(x) = f(g(x)) \longrightarrow h'(x) = g'(x)\,f'(g(x))$$
$$k(x) = g(g(x)) \longrightarrow k'(x) = g'(x)\,g'(g(x))$$

a.

$$\begin{aligned} h'(1) &= g'(1)\,f'(g(1)) \\ &= 9\,f'(4) \\ &= 9 \cdot 7 \\ &= \boxed{63} \end{aligned}$$

b.

$$\begin{aligned} h'(2) &= g'(2)\,f'(g(2)) \\ &= 7 \cdot f'(1) \\ &= 7 \cdot -6 \\ &= \boxed{-42} \end{aligned}$$

c.

$$\begin{aligned} h'(3) &= g'(3)\,f'(g(3)) \\ &= 3 \cdot f'(5) \\ &= 3 \cdot 2 \\ &= \boxed{6} \end{aligned}$$

d.

$$\begin{aligned} k'(3) &= g'(3)\,g'(g(3)) \\ &= 3 \cdot g'(5) \\ &= 3 \cdot -5 \\ &= \boxed{-15} \end{aligned}$$

e.

$$\begin{aligned} k'(1) &= g'(1)\,g'(g(1)) \\ &= 9 \cdot g'(4) \\ &= 9 \cdot -1 \\ &= \boxed{-9} \end{aligned}$$

f.

$$\begin{aligned} k'(5) &= g'(5)\,g'(g(5)) \\ &= -5 \cdot g'(3) \\ &= -5 \cdot 3 \\ &= \boxed{-15} \end{aligned}$$

4.

$$\begin{aligned} f(x) &= \sqrt{(3x-4)^2 + 3x} \\ &= \left((3x-4)^2 + 3x\right)^{1/2} \\ f'(x) &= \tfrac{1}{2}\left((3x-4)^2 + 3x\right)^{-1/2}\left[(3x-4)^2 + 3x\right]' \\ &= \tfrac{1}{2}\left((3x-4)^2 + 3x\right)^{-1/2}\left(2(3x-4)[3x-4]' + 3\right) \\ &= \boxed{\tfrac{1}{2}\left((3x-4)^2 + 3x\right)^{-1/2}\left(6(3x-4) + 3\right)} \end{aligned}$$

SINCE #4 HAD A TYPO, HERE'S AN ALTERNATIVE SOLUTION

4. (Alternative)

$$\frac{d}{dx}\left[f(x)\sqrt{(3x-4)^2+3x}\right]$$

$$= f'(x)\sqrt{(3x-4)^2+3x} + f(x)\left[\sqrt{(3x-4)^2+3x}\right]'$$

$$= \boxed{f'(x)\sqrt{(3x-4)^2+3x} + \frac{1}{2}f(x)\left((3x-4)^2+3x\right)^{-1/2}\left(6(3x-4)+3\right)}$$