Section 2.4

#3 FOR $\pi/2 < \theta < \pi$, $\sin(\theta) > 0$, AND
FOR $\pi/2 < \theta < \pi$ $\cos(\theta) < 0$. So,

$$\lim_{\theta \to \pi/2^+} \tan\theta = \lim_{\theta \to \pi/2^+} \frac{\sin\theta}{\cos\theta} = -\infty$$

Section 2.5

#2

$$f(x) = 4x\left(3x - \sqrt{9x^2+1}\right)$$
$$= \frac{4x\left(3x - \sqrt{9x^2+1}\right)}{1} \cdot \frac{\left(3x + \sqrt{9x^2+1}\right)}{\left(3x + \sqrt{9x^2+1}\right)}$$
$$= \frac{4x\left(9x^2 - 9x^2 - 1\right)}{3x + \sqrt{9x^2+1}}$$
$$= \frac{-4x}{3x + \sqrt{9x^2+1}}$$

NOTICE THAT $\lim_{x \to \pm\infty} \sqrt{9x^2+1} = \lim_{x \to \infty} 3x$, OR

LESS FORMALLY, $\sqrt{9x^2+1}$ BEHAVES LIKE $3x$ FOR SUFFICIENTLY LARGE x, AND $-3x$ FOR SUFFICIENTLY SMALL x. So

$$\lim_{x \to \infty} \frac{-4x}{3x + \sqrt{9x^2+1}} = \lim_{x \to \infty} -\frac{4x}{6x} = -\frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{-4x}{3x + \sqrt{9x^2+1}} = \lim_{x \to -\infty} \frac{-4x}{0} = \infty.$$

$y = -\frac{2}{3}$ IS THE ONLY HORIZONTAL ASYMPTOTE.

Section 2.6

#1

$$f(x) = \frac{4x^2-7}{8x^2+5x+2} \cdot \frac{1/x^2}{1/x^2}$$

$$= \frac{4-7/x^2}{8+5/x+2/x^2}$$

So

$$\lim_{x\to\infty} \frac{4-7/x^2}{8+5/x+2/x^2} = \frac{4-0}{8+0+0} = \frac{4}{8} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{4-7/x^2}{8+5/x+2/x^2} = \frac{4-0}{8+0+0} = \frac{4}{8} = \frac{1}{2}$$

#4

NOTICE $|1-x^2| = \begin{cases} 1-x^2, & -1 \le x \le 1 \\ x^2-1, & \text{OTHERWISE} \end{cases}$

a

$$\lim_{x\to\infty} \frac{|1-x^2|}{x(x+1)} = \lim_{x\to\infty} \frac{x^2-1}{x(x+1)} = \lim_{x\to\infty} \frac{x-1}{x}$$

$$= \lim_{x\to\infty} \frac{1-1/x}{1} = \frac{1-0}{1} = 1$$

$$\lim_{x\to-\infty} \frac{|1-x^2|}{x(x+1)} = \lim_{x\to-\infty} \frac{x^2-1}{x(x+1)} = \lim_{x\to\infty} \frac{x-1}{x}$$

$$= \lim_{x\to\infty} \frac{1-1/x}{1} = \frac{1-0}{1} = 1$$

b IF WE HAVE A VERTICAL ASYMPTOTE, IT'S AT $x=0$ OR $x=-1$. SO WE USE THE FIRST CASE.

$$\lim_{x\to a} \frac{|1-x^2|}{x(x+1)} = \lim_{x\to a} \frac{(1-x)(1+x)}{x(x+1)} = \lim_{x\to a} \frac{1+x}{x}$$

ASYMPTOTE AT $x=0$. SO

$$\lim_{x\to 0^+} \frac{1+x}{x} = \infty$$

$$\lim_{x\to 0^-} \frac{1+x}{x} = -\infty$$