Recitation 11: Linear Approximation & Mean Value Theorem

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Derivatives can be used to approximate values for functions that don't have nice integer answers. Consider $\sqrt{146}$. Off-hand, it's not obvious what this is, other than it sitting somewhere between 12 and 13. So, we use linear approximation to figure it out. Let

$$f(x) = \sqrt{x}.$$

Then

$$f'(x) = \frac{1}{2\sqrt{x}}.$$

Since $\sqrt{144} = 12$ and 144 is pretty close to 146, we'll take the tangent line to $f(x)$ at the point where $x = 144$ for our approximation.

At this point, finding the equation of the tangent line should be second nature - it is $y = \frac{x}{24} + 6$. When $x = 146$, we get

$$y = 12.08\bar{3}.$$

A quick check with Ye Olde calculator shows

$$\sqrt{146} = 12.083046\ldots,$$

so our approximation was pretty close. The closer we choose our point x, the more accurate our approximation.

Example (§4.5, #4)**.** Consider the function $f(x) = e^{2x}$ and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)dx$.

Solution

We have that $f'(x) = 2e^{2x}$, so $dy = 2e^{2x}dx$.

Theorem (Mean Value Theorem). *If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Geometrically, this can be interpretted as saying, for any curve on the interval $[a, b]$, there is a point on the curve whose tangent line is parallel to the secant line from a to b.

Assignment

MAT270 Recitation Notebook
§4.5, Problems 1,2
§4.6, Problems 1,4,6