

# MAT270 Exam 3 Review

Fall 2013

*This is in no way a complete list of topics covered in class, but merely a compilation of the types of exercises commonly encountered.*

1. Use the various first and second derivative techniques (intervals of increase/decrease, intervals of concavity) to make a complete graph of the functions on their domains or in the given interval.

a.  $f(x) = \frac{x^4}{2} - 3x^2 + 4x + 1$

b.  $f(x) = \frac{x^2 + x}{4 - x^2}$

c.  $f(x) = \frac{\cos(\pi x)}{1 + x^2}$  on  $[-2, 2]$ .

2. Optimization.

a. A right triangle has legs of length  $h$  and  $r$  and a hypotenuse of length 4. It is revolved about the leg of length  $h$  to sweep out a right circular cone. What values of  $h$  and  $r$  maximize the volume of the cone?

b. What two nonnegative (so, possibly zero) real numbers  $a$  who sum is 23 maximize  $a^2 + b^2$ ? Minimize  $a^2 + b^2$ ?

c. What point of the graph of  $f(x) = \frac{5}{2} - x^2$  is closest to the origin  $(0, 0)$ ? (*Hint: you can minimize the square of the distance*)

3. State Rolle's Theorem.

4. State the Mean Value Theorem.

5. Evaluate the following limits. Use L'Hôpital's Rule when needed.

a.  $\lim_{t \rightarrow 2} \frac{t^3 - t^2 - 2t}{t^2 - 4}$

b.  $\lim_{\theta \rightarrow 0} 2\theta \cot(3\theta)$

c.  $\lim_{\gamma \rightarrow 0} \frac{e^{-2x} - 1 + 2x}{x^2}$

d.  $\lim_{x \rightarrow 0^+} \frac{\ln^{10}(y)}{\sqrt{y}}$

e.  $\lim_{x \rightarrow \infty} \frac{5x^2 + 2x - 5}{\sqrt{x^4 - 1}}$

6. Determine the following indefinite integrals.

a.  $\int (x^8 - 3x^3 + 1) dx$

b.  $\int 2 \sec^2(x) dx$

c.  $\frac{12}{x} dx$

d.  $\frac{1 + \tan(\theta)}{\sec(\theta)} d\theta$

7. Find the function with the following properties:

a.  $f'(x) = 3x^2 - 1$  and  $f(0) = 10$

b.  $g'(t) = \sin(t) + 2t$  and  $g(0) = 5$

8. A rocket is launched vertically upward with an initial velocity of 120 m/s from a platform that is 125 m above the ground. Assume that the only force at work is gravity. Determine and graph the velocity and position functions of the rocket for  $t \geq 0$ .

9. For each of the following integrals, write the equations for left, middle, and right Riemann sums for an arbitrary partition  $n$ .

a.  $\int_{-2}^2 (3x^4 - 2x + 1) dx$

b.  $\int_0^1 \frac{1}{\sqrt{4 - x^2}} dx$

c.  $\int_0^{\pi/2} \sin(\theta) d\theta$

10. Evaluate the following integrals.

a.  $\int_{-2}^2 (3x^4 - 2x + 1) dx$

b.  $\int_0^{\pi/2} \sin(\theta) d\theta$

c.  $\int y^2(3y^3 + 1)^4 dy$

d.  $\int \cos(3x) dx$