MAT270 Exam 1 Review

Fall 2013

This is in no way a complete list of topics covered in class, but merely a compilation of the types of exercises commonly encountered.

1. Use the graph of f in the figure to the right to find the following values, if possible.

a. $f(-1)$

b. $\lim_{x \to -1^-} f(x)$

c. $\lim_{x \to -1^+} f(x)$

d. $\lim_{x \to -1} f(x)$

e. $f(1)$

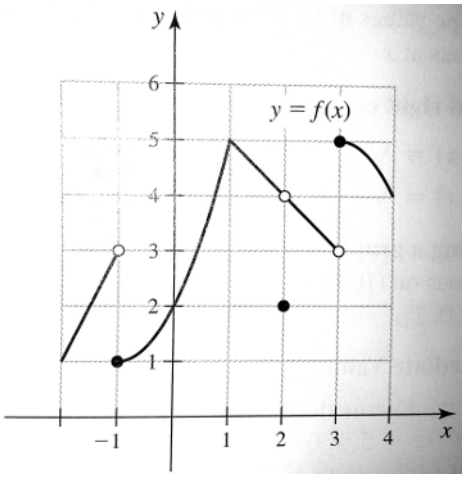
f. $\lim_{x \to 1} f(x)$

g. $\lim_{x \to 2} f(x)$

h. $\lim_{x \to 3^-} f(x)$

i. $\lim_{x \to 3^+} f(x)$

j. $\lim_{x \to 3} f(x)$



2. Calculate the following limits analytically.

a. $\lim_{x \to 1} \dfrac{x^3 - 7x^2 + 12x}{4 - x}$

b. $\lim_{x \to 4} \dfrac{x^3 - 7x^2 + 12x}{4 - x}$

c. $\lim_{p \to 1} \dfrac{p^5 - 1}{p - 1}$

d. $\lim_{x \to 3} \dfrac{x^4 - 81}{x - 3}$

e. $\lim_{\theta \to \pi/4} \dfrac{\sin^2(\theta) - \cos^2(\theta)}{\sin(\theta) - \cos(\theta)}$

f. $\lim_{x \to \pi/2} \dfrac{\frac{1}{\sqrt{\sin(x)}} - 1}{x + \pi/2}$

3. State the Squeeze Theorem.

4. Use the Squeeze Theorem to compute the following limits.

a. $\lim_{x\to 0} x\sin(x)$ [Hint: $-1 \le \sin(x) \le 1$ for all x.]

b. $\lim_{x\to\infty} \dfrac{2-\cos(x)}{x+3}$ [Hint: $1 \le 2-\cos(x) \le 3$ for all x.]

5. If possible, determine the following infinite limits.

a. $\lim_{x\to -5^+} \dfrac{x-5}{x+5}$

b. $\lim_{x\to -5^-} \dfrac{x-5}{x+5}$

c. $\lim_{x\to -5} \dfrac{x-5}{x+5}$

6. Find the horizontal asymptote(s) of each of the following functions, if they exist.

a. $f(x) = \dfrac{x^4+2}{x^5+2}$

b. $f(x) = -3x^3+5$

c. $f(x) = \dfrac{1}{\ln(x)+1}$

d. $f(x) = \sinh(x) = \dfrac{e^x - e^{-x}}{2}$

7. State the Intermediate Value Theorem.

8. Determine whether or not each of the following functions is continuous at the point given.

a. $f(x) = \dfrac{1}{x-5}$; at $x = 5$

b. $f(x) = \dfrac{x^2-25}{x-5}$; at $x = 5$

c. $f(x) = \dfrac{3x^2+2x+1}{x-1}$; at $x = 2$

d. $f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$; at $x = 0$

[Hint: Use the Squeeze Theorem to determine the limit of $f(x)$ as $x \to 0$.]

9. State the formal ε-δ definition of a limit of a function.

10. Give a formal proof of each of the following limits.

a. $\lim_{x\to 1}(5x-2) = 3$

b. $\lim_{x\to 5} \dfrac{x^2-25}{x-5} = 10$

11. Find the slope of the tangent line of the following functions at the given points.

a. $f(x) = 4x^2-1$; at $x = 4$

b. $f(x) = x^3+3$; at $x = -3$

c. $f(x) = \dfrac{1}{x}$; at $x = 1$