1. **a.** Just before the second injection, the residual concentration from the first injection is De^{-aT}. Just before the third injection, the residual concentration from the second injection is De^{-aT} and from the first injection is De^{-2aT} for a total of $De^{-aT} + De^{-2aT}$. Following this logic, the residual concentration before the $(n+1)^{\text{st}}$ injection is

$$De^{-aT} + De^{-2aT} + De^{-3aT} + \cdots + De^{-naT} = \sum_{k=1}^{n} De^{-naT}$$

which is a geometric sum with first term De^{-aT} and common ratio e^{-aT}. Thus

$$\sum_{k=1}^{n} De^{-aTn} = \frac{De^{-aT}\left(1 - e^{-(n+1)aT}\right)}{1 - e^{-aT}}.$$

b. The preinjection concentration is thus

$$\sum_{n=1}^{\infty} De^{-naT} = \frac{De^{-aT}}{1 - e^{-aT}}.$$

2. **a.** We compute the downward distance and the upward distance separately. The ball first falls from a height of H. After the first bounce, the ball reaches a maximum height of rH; after the second bounce, the ball reaches a maximum height of $r(rH) = r^2H$. The total distance the ball travels downward (assuming it bounces indefinitely) is

$$\sum_{n=0}^{\infty} Hr^n = \frac{H}{1-r}.$$

The ball travels all of the same distances upward with the exception of the initial distance H, and so the total distance the ball travels upward (assuming it bounces indefinitely) is

$$\sum_{n=1}^{\infty} Hr^n = \frac{rH}{1-r}.$$

Thus the total distance is

$$\frac{H + rH}{1-r}.$$

b. The ball falls $\frac{1}{2}gt^2$ meters in t seconds. Setting this equal to m, we can see that it takes $t = \sqrt{\frac{2m}{g}}$ seconds for the ball to move m verticle meters. Thus the total time (in seconds) the ball spent moving the distance above is

$$\sum_{n=0}^{\infty} \sqrt{\frac{2r^nH}{g}} + \sum_{n=1}^{\infty} \sqrt{\frac{2r^nH}{g}} = \sum_{n=0}^{\infty} (\sqrt{r})^n \sqrt{\frac{2H}{g}} + \sum_{n=1}^{\infty} (\sqrt{r})^n \sqrt{\frac{2H}{g}}$$
$$= \left(\frac{1}{1-\sqrt{r}}\right)\left(\sqrt{\frac{2H}{g}} + \sqrt{\frac{2rH}{g}}\right).$$

3. **a.** We have that $a_n = \frac{n}{5^n}$ and $a_{n+1} = \frac{n+1}{5^{n+1}}$. Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{5^{n+1}}\cdot\frac{5^n}{n}\right| = \lim_{n\to\infty}\left|\frac{1}{5}\cdot\frac{n+1}{n}\right| = \frac{1}{5}.$$

By the ratio test, this series is absolutely convergent.

b. We have that $a_n = \frac{(-3)^n}{(2n+1)!}$ and $a_{n+1} = \frac{(-3)^{n+1}}{(2n+3)!}$. Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-3)^{n+1}}{(2n+3)!}\cdot\frac{(2n+1)!}{(-3)^n}\right| = \lim_{n\to\infty}\left|\frac{3}{(2n+3)(2n+2)}\right| = 0 < 1.$$

By the ratio test, this series is absolutely convergent.

c. We have that $a_n = n\left(\frac{2}{3}\right)^n$ and $a_{n+1} = (n+1)\left(\frac{2}{3}\right)^{n+1}$. Thus

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(2/3)^{n+1}}{n(2/3)^n}\right| = \lim_{n\to\infty}\left|\frac{2(n+1)}{3n}\right| = \frac{2}{3} < 1.$$

By the ratio test, this series is absolutely convergent.

4. Applying the ratio test with $a_n = \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$ and $a_{n+1} = \frac{(-1)^{n+1}}{(n+1)!(n+2)!2^{2n+3}}$ we have

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2n+3}}{(n+1)!(n+2)!2^{2n+3}}\cdot\frac{n!(n+1)!2^{2n+1}}{(-1)^n x^{2n+1}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{4(n+2)(n+1)}\right| = 0 < 1.$$

Thus the series converges for all real numbers x, and so the interval of convergence is $(-\infty, \infty)$.

5. Notice that the function can be written succinctly as

$$A(x) = \sum_{n=0}^{\infty} x^{3n}(3n)!$$

Applying the ratio test with $a_n = x^{3n}(3n)!$ and $a_{n+1} = x^{3n+3}(3n+3)!$ we have

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{3n+3}}{(3n+3)!}\cdot\frac{(3n)!}{x^{3n}}\right| = \lim_{n\to\infty}\left|\frac{x^3}{(3n+3)(3n+2)(3n+1)}\right| = 0 < 1.$$

Thus the series converges for all real numbers x, and so the interval of convergence is $(-\infty, \infty)$.