Quiz Date: November 01, 2018

Instructions: The following exercises are similar to those found in the course text book. This homework is not due for a grade, but you should know how to do all of the exercises and be able to show your work for each. You can expect at least one of these problems to appear on an in-class quiz on the date listed above.

## 8.2 - Series

1. After injection of a dose $D$ of insulin, the concentration of insulin in a patient's system decays exponentially and so it can be written as $D e^{-a t}$ where $t$ represents the time in hours and $a$ is a positive constant.
a. If a dose $D$ is injected every $T$ hours, write an expression for the sum of the residual concentrations just before the $(n+1)^{\text {st }}$ injection.
b. Determine the pre-injection concentration.
2. A certain ball has the property that each time it falls from a height $h$ onto a hard, level surface, it rebounds to a height of $r h$ where $0<r<1$. Suppose the ball is dropped from an initial height of $H$ meters.
a. Assuming the ball continues to bounce indefinitely, find the total vertical distance that it travels.
b. Calculate the total time the ball travels. (Use the fact that the ball falls $\frac{1}{2} g t^{2}$ meters in $t$ seconds.)

## 8.4 - Other Convergence Tests

3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.
a. $\sum_{n=1}^{\infty} \frac{n}{5^{n}}$
b. $\sum_{n=0}^{\infty} \frac{(-3)^{n}}{(2 n+1)!}$
c. $\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^{k}$

## 8.5 - Power Series

4. The function $J_{1}$ defined by

$$
J_{1}(x)=\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{2 n+1}}{n!(n+1)!2^{2 n+1}}
$$

is called the Bessel function of order 1. It arises naturally as the solution to certain differential equation. Find its interval of convergence.
5. The function $A$ defined by

$$
A(x)=1+\frac{x^{3}}{2 \cdot 3}+\frac{x^{6}}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}+\frac{x^{9}}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}+\cdots
$$

is called an Airy function after English mathematicians and astronomer Sir George Airy. It arises as the solution to a certain differential equation. Find its interval of convergence.

