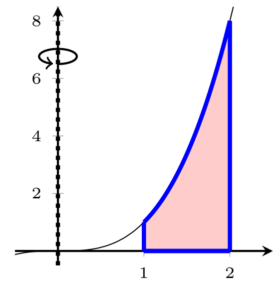
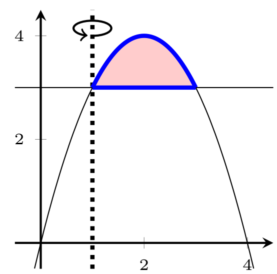
1. We use the shell method, and so integrate with respect to x. At each fixed x, the cylindrical shell has radius $r = x$ and height $h = x^3$, and so the volume of the solid is

$$
\begin{aligned}
V &= \int_1^2 2\pi r h \, dx \\
&= 2\pi \int_1^2 x \cdot x^3 \, dx \\
&= 2\pi \int_1^2 x^4 \, dx \\
&= \pi \left[\frac{2}{5} x^5 \right]_1^2 \\
&= \boxed{\frac{62}{5}\pi \approx 38.95575}
\end{aligned}
$$



2. We use the shell method, and so integrate with respect to x. At each x, the cylindrical shell has radius $r = (x - 1)$ and height $h = (4x - x^2) - 3$, and so the volume of the solid is

$$
\begin{aligned}
V &= \int_1^3 2\pi r h \, dx \\
&= 2\pi \int_1^3 (x-1)(4x - x^2 - 3) \, dx \\
&= 2\pi \int_1^3 -x^3 + 5x^2 - 7x + 3 \, dx \\
&= 2\pi \left[-\frac{1}{2} x^3 + \frac{5}{3} x^3 - \frac{7}{2} x + 3x \right]_1^3 \\
&= \boxed{\frac{8}{3}\pi \approx 8.37758}
\end{aligned}
$$



3. We first note the following (which can be found in the table of integrals or with a combination of trig substitution and integration by parts).

$$
\boxed{\int \sqrt{u^2 + a^2} \, du = \frac{a^2}{2} \ln \left| \sqrt{a^2 + u^2} + u \right| + \frac{u}{2} \sqrt{a^2 + u^2} + C}
$$

Now,

$$
1 + (y')^2 = 1 + \frac{1}{400}(x - 50)^2 = \left(\frac{1}{20} \right)^2 \left[20^2 + (x - 50)^2 \right],
$$

and thus the distance the kite traveled from $x = 0$ to $x = 80$ is

$$
\begin{aligned}
\mathcal{L} &= \int_0^{80} \sqrt{1 + (y')^2} \, dx \\
&= \int_0^{80} \sqrt{\left(\frac{1}{20} \right)^2 [20^2 + (x - 50)^2]} \, dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{20}\int_0^{80} \sqrt{20^2 + (x-50)^2}\,dx \\
&= \frac{1}{20}\left[\frac{400}{2}\ln\left|\sqrt{400+(x-50)^2} + (x-50)\right| + \frac{(x-50)}{2}\sqrt{400+(x-50)^2}\right]_0^{80} \\
&\approx 122.77614\,\text{ft}
\end{aligned}
$$

4. The bird lets go at $x = 0\,\text{m}$ (the max of the parabola) and hits the ground when $x = 90\,\text{m}$. Now, we have that

$$
1 + (y')^2 = 1 + \left(\frac{2}{45}\right)^2 x^2 = \left(\frac{2}{45}\right)^2 \left[\left(\frac{45}{2}\right)^2 + x^2\right],
$$

and thus the distance traveled by the prey is

$$
\begin{aligned}
\mathcal{L} &= \int_0^{90} \sqrt{1 + (y')^2}\,dy \\
&= \int_0^{90} \sqrt{\left(\frac{2}{45}\right)^2 \left[\left(\frac{45}{2}\right)^2 + x^2\right]}\,dx \\
&= \int_0^{90} \frac{2}{45}\sqrt{\left(\frac{45}{2}\right)^2 + x^2\,dx} \\
&= \frac{2}{45}\left[\frac{2}{45^2}\ln\left|\sqrt{\left(\frac{45}{2}\right)^2 + x^2} + x\right| + \frac{x}{2}\sqrt{\left(\frac{45}{2}\right)^2 + x^2}\right]_0^{90} \\
&\approx 209.10527\,\text{m}
\end{aligned}
$$

5. This amounts to finding the arc length of the function $y = \sin(\pi x/7)$ for $0 \le x \le 28$. Since $y' = \frac{\pi}{7}\cos(\pi x/7)$, we get that the arc length w is given by the integral

$$
w = \int_0^{28} \sqrt{1 + (y')^2}\,dx = \int_0^{28} \sqrt{1 + \frac{\pi^2}{49}\cos^2\left(\frac{\pi x}{7}\right)}\,dx
$$

The indefinite integral does not have a closed form in terms of elementary functions, but with numerical techniques we get that $w \approx 29.3607\,\text{in}$.

6. We place the bottom of the tank at the origin. A vertical cross-sectional slice of the paraboloid is the parabola $y = x^2/4$, and so at each height y, the slice of water is a circle of radius $\sqrt{4y}$. Thus, the volume of each infinitesimal slice is $V(y) = 4\pi y\,dy$.

a. The work done in emptying the tank is

$$\begin{aligned} W &= \int_0^4 \rho g(4-y)V(y) \\ &= 4\pi(62.5)\int_0^4 4y - y^2\,dy \\ &= 250\pi\left[2y^2 - \frac{1}{3}y^3\right]_0^4 \\ &= \frac{8000\pi}{3} \approx 8377.6\,\text{ft-lb} \end{aligned}$$

b. After 4000 ft-lb of work has been done, h feet of water remain in the tank. Since we're draining the water from the top of the tank, the work done in draining this is

$$\begin{aligned} 4000 &= 250\pi\int_h^4 4y - y^2\,dy \\ &= -\frac{250}{3}\pi(h-4)^2(h+2) \end{aligned}$$

Solving for h (and noting that $0 < h < 4$), we get that there is only $h \approx 2.06011\,\text{ft}$ of water left in the tank.