1. To find the position $s(t)$ we'll need to integrate $v(t)$, so we'll make the substitution

$$u = \cos(\omega t) \qquad\qquad du = -\omega \sin(\omega t)\, dt$$

whence

$$\begin{aligned} s(t) = \int v(t)\, dt = \int \sin(\omega t)\cos^2(\omega t)\, dt \\ &= -\frac{1}{\omega}\int u^2\, du \\ &= -\frac{1}{3\omega}u^3 + C \\ &= -\frac{1}{3\omega}\cos^3(\omega t) + C. \end{aligned}$$

Since $s(0) = 0$, we have that $C = \frac{1}{3\omega}$ and thus

$$s(t) = -\frac{1}{3\omega}\cos^3(\omega t) + \frac{1}{3\omega}.$$

2. **a.** $V_{\text{RMS}} = \sqrt{\frac{1}{b-a}\int_a^b [E(t)]^2\, dt}$. Since we're told that the wave is 60 Hz (i.e. 60 cycles per second), we integrate from $a = 0$ to $b = \frac{1}{60}$. Doing just the integral first, we get

$$\begin{aligned} \frac{1}{\frac{1}{60} - 0}\int_0^{1/60} [E(t)]^2\, dt &= 60\int_0^{1/60} 155^2 \sin^2(120\pi t)\, dt \\ &= 60 \cdot 155^2 \int_0^{1/60} \frac{1}{2} - \frac{1}{2}\cos(240\pi t)\, dt \qquad \text{(power reducing formula)} \\ &= 60 \cdot 155^2 \left[\frac{1}{2}t - \frac{1}{480\pi}\sin(240\pi t)\right]_0^{1/60} \\ &= 60 \cdot 155^2 \left(\frac{1}{120}\right) = \frac{24025}{2} \end{aligned}$$

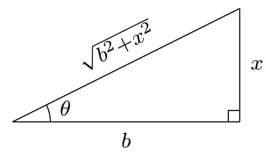
Thus

$$V_{\text{RMS}} = \sqrt{\frac{24025}{2}} = \frac{155}{\sqrt{2}}\,\text{V} \approx 109.60155\,\text{V}$$

b. Without much effort, we can see from the previous part that changing the amplitude A yields $V_{\text{RMS}} = A/\sqrt{2}$. So when $V_{\text{RMS}} = 220\,\text{V}$, we have $A = 220\sqrt{2} = 311.12698$.

3. We'll approach with a trigonometric substitution and corresponding reference triangle

$$\begin{aligned} x &= b\tan\theta \\ dx &= b\sec^2\theta\, d\theta \end{aligned}$$



The indefinite integral then becomes

$$
\begin{aligned}
E(P) = \int \frac{\lambda b}{4\pi\varepsilon_0 (x^2+b^2)^{3/2}}\,dx &= \frac{\lambda b}{4\pi\varepsilon_0}\int \frac{dx}{(x^2+b^2)^{3/2}} \\
&= \frac{\lambda b}{4\pi\varepsilon_0}\int \frac{b\sec^2\theta\,d\theta}{(b^2\tan^2\theta+b^2)^{3/2}} \\
&= \frac{\lambda b}{4\pi\varepsilon_0}\int \frac{b\sec^2\theta\,d\theta}{(b^2\sec^2\theta)^{3/2}} \\
&= \frac{\lambda b}{4\pi\varepsilon_0}\int \frac{b\sec^2\theta\,d\theta}{b^3\sec^3\theta} \\
&= \frac{\lambda}{4b\pi\varepsilon_0}\int \cos\theta\,d\theta \\
&= \frac{\lambda}{4b\pi\varepsilon_0}\sin\theta + C \\
&= \frac{\lambda}{4b\pi\varepsilon_0}\frac{x}{\sqrt{b^2+x^2}} + C
\end{aligned}
$$

Applying the fundamental theorem of calculus and evaluating the definite integral we get

$$
E(P) = \left[\frac{\lambda}{4b\pi\varepsilon_0}\frac{x}{\sqrt{b^2+x^2}}\right]_{-a}^{L-a} = \frac{\lambda}{4b\pi\varepsilon_0}\left[\frac{L-a}{\sqrt{b^2+(L-a)^2}} + \frac{a}{\sqrt{b^2+a^2}}\right].
$$

4. Skipped because it requires material from §7.1

5. Noting that $S = 900$ and $(r-1) = -0.90$ are constants, our integrand is a rational expression in P, so we can perform a partial fraction decomposition

$$
\begin{aligned}
\frac{P+900}{P[-0.90P-900]} &= \frac{A}{P} + \frac{B}{-0.90P-900} \\
P+900 &= A[-0.90P-900] + BP = [-0.90A+B]P + (-A)900
\end{aligned}
$$

which yields the following system of linear equations

$$
1 = -0.90A + B1 \qquad\qquad = -A
$$

And thus $A = -1$, $B = 0.10$. We now use the partial fraction decomposition to evaluate the integrals

$$
\begin{aligned}
t = \int \frac{P+900}{P[-0.90P-900]}\,dP &= \int \frac{-1}{P} + \frac{0.10}{-0.90P-900}\,dP \\
&\int \frac{-1}{P} + \frac{0.10}{0.90P+900}\,dP \\
&= -\ln|P| - 0.10\ln|0.90P+900| + C
\end{aligned}
$$

At time $t = 0$ we have that $P = 10000$, and we can use this to solve for our integration constant above: $C = \ln|10000| + 0.10\ln|9900|$. Thus the equation relating the female population and time is

$$
t = -\ln|P| + 0.10\ln|0.90P+900| + \ln|10000| + 0.10\ln|9900|.
$$

6. The average cost for removing between 75% and 80% is given by

$$C_{\text{avg}} = \frac{1}{80-75}\int_{75}^{80} \frac{124p}{(10+p)(100-p)}\,dp$$

We can use a partial fraction decomposition to simplify the integrand, whence

$$\begin{aligned}
C_{\text{avg}} &= \frac{1}{80-75}\int_{75}^{80} -\frac{124}{11}\frac{1}{10+p} + \frac{1240}{11}\frac{1}{100-p}\,dp \\
&= \left[-\frac{124}{11}\ln(10+p) - \frac{1240}{11}\ln(100-p)\right]_{75}^{80} \\
&= -\frac{124}{11}\ln(90) - \frac{1240}{11}\ln(10) + \frac{124}{11}\ln(85) + \frac{1240}{11}\ln(25) \approx 24.5100\,\text{currencyunits}
\end{aligned}$$