Quiz Date: August 28, 2018

Instructions: The following exercises are similar to those found in the course text book. This homework is not due for a grade, but you should know how to do all of them and be able to show your work for each. You can expect at least one of these problems to appear on an in-class quiz on the date listed above.

1 5.5 - The Substitution Rule

1. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

2. The rate of dispersement $\frac{dQ}{dt}$ of a 2-million dollar federal grant is proportional to the square of $100 - t$. Time t is measured in days ($0 \leq t \leq 100$), and Q is the amount that remains to be disbursed. Find the amound that remains to be disbursed after 50 days. Assume that all the money will be disbursed in 100 days.

3. The probability that ore samples taken from a region contain between $a\%$ and $b\%$ iron is

$$P_{a,b} = \int_a^b \frac{1155}{32} x^3 (1-x)^{3/2}\, dx$$

where x represents the percentage of iron. What is the probability that a sample will contain between

a. 0% and 25% iron?

b. 50% and 100% iron?

4. The number of sales S (in thousands of units) of a seasonal product are given by the model

$$S = 74.50 + 43.75 \sin\left(\frac{\pi t}{6}\right)$$

where t is the time in months, and $t = 1$ corresponds to January. Find the average number sales for each time period.

a. The first quarter ($0 \leq t \leq 3$).

b. The second quarter.

c. The entire year.

2 6.1 - Integration by Parts

5. A rocket accelerates by burning its onboard fuel, so its mass decreases with time. Suppose the initial mass of the rocket at liftoff (including its fuel) is m, the fuel is consumed at rate r, and the exhaust gases are ejected with constant velocity v_e (relative to the rocket). A model for the velocity of the rocket at time t (in seconds) is given by the equation

$$v(t) = -gt - v_e \ln\left(\frac{m - rt}{m}\right)$$

where g is acceleration due to gravity and t is not too large. If $g = 9.8\,\mathrm{m/s^2}$, $m = 30,000\,\mathrm{kg}$, $r = 160\,\mathrm{kg/s}$, and $v_e = 3000\,\mathrm{m/s}$, find the height of the rocket 1 minute after liftoff.

6. A particle the moves along a straight line has velocity $v(t) = t^2 e^{-t}$ meters per second after t seconds. How far will it travel during the first T seconds?

7. A string stretched between the two points $(0, 0)$ and $(2, 0)$ is plucked by displacing the string h units at its midpoint. The motion of the string is modeled by a *Fourier Sine Series* whose coefficients are given by

$$b_n = h \int_0^1 x \sin\left(\frac{n\pi x}{2}\right)\,dx + h \int_1^2 (-x + 2) \sin\left(\frac{n\pi x}{2}\right)\,dx.$$

Evaluate the integrals to find an explicit formula for b_n.