1. To find the slope, we need $\frac{dy}{dx}$. Thus

$$\begin{aligned}\frac{d}{dx}[2] &= \frac{d}{dx}\left[-x^2 + 2\sqrt{3}xy + y^2\right] \\ 0 &= -2x + 2\sqrt{3}y + 2\sqrt{3}x\frac{dy}{dx} + 2y\frac{dy}{dx} \\ 2x - 2\sqrt{3}y &= \left(2\sqrt{3}x + 2y\right)\frac{dy}{dx} \\ \Rightarrow \quad \frac{dy}{dx} &= \frac{2x - 2\sqrt{3}y}{2\sqrt{3}x + 2y}.\end{aligned}$$

So, plugging in the point $(0, -\sqrt{2})$ we have that the slope of the tangent line is $-\sqrt{3}$, whence the the tangent line is given by the equation

$$y = (\sqrt{3})x - \sqrt{2}.$$

2. **a.** Let $A = A(t)$ be the area, and $r = r(t)$ the radius at time t. We have that $A = \pi r^2$, so differentiating implicitly, we get

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}.$$

b. We're given that $\frac{dr}{dt} = 8\,\text{in/s}$. So when $r = 29\,\text{in}$, by our results from the previous part

$$\frac{dA}{dt} = 2\pi(29)(8) = 464\pi\,\text{in}^2/\text{s} \approx 1457.7\,\text{in}^2/\text{s}.$$

3. Let $A = A(t)$ be the area, $\ell = \ell(t)$ the length, and $w = w(t)$ the width. We have that $A = \ell w$, so differentiating implicitly we get that

$$\frac{dA}{dt} = w\frac{d\ell}{dt} + \ell\frac{dw}{dt}.$$

We're given that $\frac{d\ell}{dt} = 5\,\text{cm/s}$ and $\frac{dw}{dt} = -5\,\text{cm/s}$, so when $\ell = 20\,\text{cm}$ and $w = 30\,\text{cm}$, we have

$$\frac{dA}{dt} = 30(5) + 20(-5) = 10\,\text{cm}^2/\text{s}$$

When $\ell(t) = 40$ and $w(t) = 10$, we get

$$\frac{dA}{dt} = 10(5) + 40(-5) = -150,\text{cm}^2/\text{s},$$

so the area is *decreasing.*

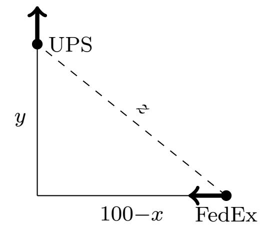
4. Let $V = V(t)$ be the volume and $r = r(t)$ the area at time t. We have that $V = \frac{4}{3}\pi r^3$, so differentiating implicitly we get

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}.$$

We're given that $\frac{dr}{dt} = 3\,\text{mm/s}$, so when the radius is $75\,\text{mm}$,

$$\frac{dV}{dt} = 4\pi[75]^2(3) = 67\,500\pi\,\text{mm}^3/\text{s} \approx 212,057.5\,\text{mm}^3/\text{s}.$$

5.



Let $t = 0$ at 12:00PM. Let $z = z(t)$ be the distance between the UPS truck and the FedEx truck at time t, $x = x(t)$ the east/west distance traveled, and $y = y(t)$ the north/south distance. By the Pythagorean theorem, we have $z^2 = x^2 + y^2$, hence

$$\frac{dz}{dt} = -\frac{(100-x)}{z}\frac{dx}{dt} + \frac{y}{z}\frac{dy}{dt}.$$

We're given that $\frac{dx}{dt} = 55\,\text{mi/h}$ and $\frac{dy}{dx} = 75\,\text{mi/h}$. So when $t = 2$ hours, $x = 100\,\text{mi}$, $y = 150\,\text{mi}$ and $z = \sqrt{(100-x)^2 + y^2} = \sqrt{22600}$, thus

$$\frac{dz}{dt} = -\frac{(100-x)}{z}\frac{dx}{dt} + \frac{y}{z}\frac{dy}{dt} = \frac{10}{\sqrt{22600}}(55) + \frac{150}{\sqrt{22600}}(75) \approx 78.49\,\text{mi/h}.$$

6. Let $V = V(t)$ be the volume, $r = r(t)$ the radius, and $h = h(t)$ the height at time t. We have that $V = \frac{\pi}{3}r^2h$. Since the ratio of the radius to the height is constantly $\frac{9}{7}$, we can actually deduce that $r = \frac{9}{7}h$ so in fact $V = \frac{3\pi}{7}h^3$. By implicit differentiation, we have that

$$\frac{dV}{dt} = \frac{9\pi}{7}h^2\frac{dh}{dt}.$$

We're given that $\frac{dV}{dt} = 10 - c\,\text{ft}^3/\text{min}$ (where c is the constant speed at which the water is leaking out of the tank) and that $\frac{dh}{dt} = -1\,\text{ft/min}$. When the height is $3\,\text{ft}$, we get

$$10 - c = \frac{9\pi}{7}(3)^2(-1) = -\frac{81\pi}{7}$$
$$\Rightarrow \quad c = 10 + \frac{81\pi}{7}\,\text{ft}^3/\text{min} \approx 46.35\,\text{ft}^3/\text{min}.$$