§2.4 The Product and Quotient Rules

1. If $f(x) = 4x^3 \sec x$, find $f'(x)$.

2. Find $f'\left(\frac{\pi}{2}\right)$ where $f(x) = \dfrac{x+1}{\sin x}$.

3. Find the second derivative of $y = \frac{x^2}{3x-7}$.

§2.5 The Chain Rule

4. Use the table below to answer parts (a)-(c).

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
10	13	12	13	11
11	10	13	11	12
12	11	10	12	13
13	12	11	10	13

   a. $J(x) = (g \circ f)(x) = g(f(x))$. What is $J'(10)$?
   b. $K(x) = (f \circ g)(x) = f(g(x))$. What is $K'(10)$?
   c. $M(x) = (J \cdot K)(x) = J(x)K(x)$. What is $M'(10)$?

5. Find the 45th derivative of $y = \sin(3x)$.

6. Determine $\frac{dy}{dt}$ given $y = \cot(\sqrt{t})$

§2.6 Implicit Differentiation

7. Find the slope of the tangent line to the curve $7x^2y^2 - 3y^4 = -20$ at $(1, 2)$.

8. Find the slope of the normal line to the curve $y^5 + 4x^3y = 5$ at $(1, 1)$.

9. For the given equation, evaluate y' at the point $(-1, -1)$:

$$(2x^2 + y)^4 + y = 0.$$

§2.7 Related Rates

10. Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius in ft/s when the radius is 0.5 feet. Recall that the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

11. The volume of a cylinder in increasing at a rate of 15 cubic meters per minute. If the radius is constantly half of the height, find the rate of change of the height of the cylinder when the radius is 5 meters. Recall that the volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$.

§2.8 Linear Approximations and Differentials

12. Write the linear approximation $L(x)$ at $x = \frac{\pi}{2}$ of the function $f(x) = \sqrt{9 - 9\cos x}$.

13. Use linear approximation to approximate $\sqrt{144.1}$. Write your answer to 5 decimal places.

§3.1 Exponential Functions

14. Find the following limits exactly:

    a. $\lim\limits_{x \to 5^+} e^{2/(x-5)}$

    b. $\lim\limits_{t \to -\infty} (1.000001)^t$

    c. $\lim\limits_{k \to 0} (1+k)^{1/k}$

    d. $\lim\limits_{x \to \infty} \dfrac{18(5)^x - 17}{3(5)^x + 41}$

§3.2 Inverse Functions and Logarithms

15. Let $f(x) = \frac{1}{4}x^3 + x - 1$. Find the derivative of the inverse function of f at $c = 3$, where $a = f^{-1}(c)$. In other words, find $(f^{-1})'(c)$.

16. Find the limit exactly: $\lim\limits_{x \to \infty} \ln(81x - 49) - \ln(3x + 187.5)$.

17. Use properties of logarithms to rewrite the following as a single logarithm

$$\ln(x^4 + 7) - 5\ln(\sin x) + \frac{1}{2}\ln x - \ln(\sqrt{x} - 9) + \ln 1 + e^{\ln(\ln x)}.$$

§3.3 Derivatives of Logarithmic and Exponential Functions

18. Use logarithmic differentiation to find $f'(x)$ where $f(x) = \dfrac{x^3\sqrt{x^2 + 9x - 7}}{e^{3x}\cos(14x^3)}$.

19. Evaluate $\dfrac{d}{dy}\left[(\ln y^3)^5\right]$.

20. Find the second derivative of $y = \ln(\ln x)$.

§3.5 Inverse Trigonometric Functions

21. Find $\frac{dy}{dx}$ when $y = (\log_2 x)\arcsin x$

22. Let $f(x) = 2\tan^{-1}(4^x)$. Find $f'(x)$.

23. Find the equation of the tangent line to $y = \cos^{-1}(3x) + \frac{5\pi}{6}$ when $x = \frac{1}{\sqrt{12}}$.

§3.7 Indeterminate Forms and L'Hôpital's Rule

Evaluate the following limits:

24. $\lim\limits_{\theta \to 0} \dfrac{1}{\theta \cot \theta}$.

25. $\lim\limits_{x \to \infty} \left(1 + \dfrac{a}{x}\right)^{bx}$, where a, b are fixed real numbers.

26. $\lim\limits_{t \to \pi/2} \sec(3x)\cos(5x)$.

27. $\lim\limits_{x \to 0^+} \dfrac{\ln|\ln x|}{x}$.

1. $f'(x) = 12x^2 \sec x + 4x^3 \sec x \tan x$

2. $f'\left(\dfrac{\pi}{2}\right) = 1$

3. $y'' = \dfrac{98}{(3x-7)^3}$

4.   a. $J'(10) = 169$
     b. $K'(10) = 132$
     c. $M'(10) = 3311$

5. $y^{(45)} = 3^{45} \cos(3x)$

6. $\dfrac{dy}{dt} = -\dfrac{\csc^2(\sqrt{t})}{2\sqrt{t}}$

7. $y = \dfrac{14}{17}x + \dfrac{20}{17}$

8. $y = \dfrac{3}{4}x + \dfrac{1}{4}$

9. $\dfrac{16}{5}$

10. $\dfrac{dr}{dt} = \dfrac{4.5}{60\pi}\,\text{ft/s} \approx 0.024\,\text{ft/s}$

11. $\dfrac{dh}{dt} = \dfrac{1}{5\pi}\,\text{m/min} \approx 0.064\,\text{m/min}$

12. $L(x) = \dfrac{3}{2}x - \dfrac{3\pi}{4} + 3$

13. $\sqrt{144.1} \approx 12.00417$

14.   a. ∞
     b. 0
     c. e
     d. 6

15. $\dfrac{1}{4}$

16. $\ln 27 = 3\ln 3$

17. $\ln\left(\dfrac{(x^4+7)x^{3/2}}{(\sin^5 x)(\sqrt{x}-9)}\right)$

18. $f'(x) = \dfrac{x^3\sqrt{x^2+9x-7}}{e^{3x}\cos(14x^3)}\left(\dfrac{3}{x} + \dfrac{1}{2}\cdot\dfrac{2x+9}{x^2+9x-7} - 3 + \dfrac{\sin(14x^3)\cdot 42x^2}{\cos(14x^3)}\right)$

19. $\dfrac{15\left(\ln y^3\right)^4}{y}$

20. $\dfrac{\ln x + 1}{-(x\ln x)^2}$

21. $\dfrac{dy}{dx} = \dfrac{\arcsin x}{x\ln 2} + \dfrac{\log_2 x}{\sqrt{1-x^2}}$

22. $f'(x) = \dfrac{2\cdot 4^x \ln 4}{1+4^{2x}} = \dfrac{4^{x+1}\ln 2}{1+16^x}$

23. $y = -6x + \pi + \sqrt{3}$

24. 1

25. e^{ab}

26. $-\dfrac{5}{3}$

27. ∞