# Chapter 5 Review Exercises

**2.** Working on the left side,

$$\begin{aligned}
\cos x + \sin x \tan x &= \cos x + \frac{\sin^2 x}{\cos x} && \text{(quotient identity)}\\
&= \frac{\cos^2 x}{\cos x} + \frac{\sin^2}{\cos x}\\
&= \frac{\cos^2 x + \sin^2 x}{\cos x}\\
&= \frac{1}{\cos x} && \text{(Pythagorean identity)}\\
&= \sec x. && \text{(reciprocal identity)}
\end{aligned}$$

**4.** Working on the left side,

$$\begin{aligned}
(\sec\theta - 1)(\sec\theta + 1) &= \sec^2\theta - 1\\
&= \left(1 + \tan^2\theta\right) - 1 && \text{(Pythagorean identity)}\\
&= \tan^2\theta.
\end{aligned}$$

**6.** Working on the left side,

$$\begin{aligned}
\frac{1}{\sin t - 1} + \frac{1}{\sin t + 1} &= \frac{\sin t + 1}{(\sin t - 1)(\sin t + 1)} + \frac{\sin t - 1}{(\sin t + 1)(\sin t - 1)}\\
&= \frac{\sin t + 1}{\sin^2 t - 1} + \frac{\sin t - 1}{\sin^2 t - 1}\\
&= \frac{(\sin t + 1) + (\sin t - 1)}{\sin^2 t - 1}\\
&= \frac{(\sin t + 1) + (\sin t - 1)}{\cos^2 t} && \text{(Pythagorean identity)}\\
&= \frac{2\sin t}{\cos^2 t}\\
&= 2\frac{\sin t}{\cos t}\left(\frac{1}{\cos t}\right)\\
&= 2\tan t\left(\frac{1}{\cos t}\right) && \text{(quotient identity)}\\
&= 2\tan t\sec t. && \text{(reciprocal identity)}
\end{aligned}$$

**8.** Working on the left side,

$$\begin{aligned}
\frac{\cos x}{1 - \sin x} &= \frac{\cos x}{1 - \sin x}\left(\frac{1 + \sin x}{1 + \sin x}\right)\\
&= \frac{\cos x\,(1 + \sin x)}{1 - \sin^2 x}\\
&= \frac{\cos x\,(1 + \sin x)}{\cos^2 x} && \text{(Pythagorean identity)}\\
&= \frac{1 + \sin x}{\cos x}. && \text{(Pythagorean identity)}
\end{aligned}$$

**10.** Starting with the left-hand side of the equation,

$$\begin{aligned}
(\tan\theta + \cot\theta)^2 &= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 && \text{(quotient identity)}\\
&= \left(\frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}\right)^2\\
&= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2\\
&= \left(\frac{1}{\sin\theta\cos\theta}\right)^2 && \text{(Pythagorean identity)}\\
&= \frac{1}{\sin^2\theta\cos^2\theta},
\end{aligned}$$

and the right-hand side of the equation becomes

$$\begin{aligned}
\sec^2\theta + \csc^2\theta &= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} && \text{(reciprocal identity)}\\
&= \frac{\sin^2\theta}{\cos^2\theta\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta}\\
&= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}\\
&= \frac{1}{\sin^2\theta\cos^2\theta}. && \text{(Pythagorean identity)}
\end{aligned}$$

**12.** Working on the left side, we have

$$\begin{aligned}
\frac{\cos t}{\cot t - 5\cos t} &= \frac{\cos t}{\frac{\cos t}{\sin t} - 5\cos t} && \text{(quotient identity)}\\
&= \frac{\cos t}{\cos t\left(\frac{1}{\sin t} - 5\right)}\\
&= \frac{1}{\frac{1}{\sin t} - 5}\\
&= \frac{1}{\csc t - 5}. && \text{(reciprocal identity)}
\end{aligned}$$

**14.**

$$\begin{aligned}
\cos(45^\circ + 30^\circ) &= \cos(45^\circ)\cos(30^\circ) - \sin(45^\circ)\sin(40^\circ)\\
&= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)\\
&= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\
&= \frac{\sqrt{6} - \sqrt{2}}{4}.
\end{aligned}$$

**16.**

$$\begin{aligned}\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right) &= \frac{\tan\left(\frac{4\pi}{3}\right)-\tan\left(\frac{\pi}{4}\right)}{1+\tan\left(\frac{4\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}\\ &= \frac{\sqrt{3}-1}{1+\sqrt{3}}\\ &= 2-\sqrt{3}.\end{aligned}$$

**18.**

$$\begin{aligned}\cos(65^\circ)\cos(5^\circ)+\sin(65^\circ)\sin(5^\circ) &= \cos(65^\circ-5^\circ)\\ &= \cos(60^\circ)\\ &= \frac{1}{2}.\end{aligned}$$

**20.** Working with the left side, we have

$$\begin{aligned}&\sin\left(x+\frac{\pi}{6}\right)-\cos\left(x+\frac{\pi}{3}\right)\\ &= \sin x\cos\left(\frac{\pi}{6}\right)+\cos x\sin\left(\frac{\pi}{6}\right)-\cos x\cos\left(\frac{\pi}{3}\right)+\sin x\sin\left(\frac{\pi}{3}\right) \qquad \text{(Angle-Sum formulas)}\\ &= \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x-\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\\ &= \sqrt{3}\sin x.\end{aligned}$$

**24.** Working with the left side, we have

$$\begin{aligned}\cos^4 t-\sin^4 t &= \left(\cos^2 t+\sin^2 t\right)\left(\cos^2 t-\sin^2 t\right)\\ &= \cos^2 t-\sin^2 t && \text{(Pythagorean identity)}\\ &= \cos(2t) && \text{(Double-Angle formula).}\end{aligned}$$

**26.** Starting on the left side, we have

$$\begin{aligned}\frac{\sin(2\theta)-\sin\theta}{\cos(2\theta)+\cos\theta} &= \frac{2\sin\theta\cos\theta-\sin\theta}{2\cos^2\theta-1+\cos\theta} && \text{(Double-Angle formula)}\\ &= \frac{\sin\theta(2\cos\theta-1)}{(2\cos\theta-1)(1+\cos\theta)}\\ &= \frac{\sin\theta}{1+\cos\theta},\end{aligned}$$

and working on the right side, we have

$$\begin{aligned}\frac{1-\cos\theta}{\sin\theta} &= \frac{1-\cos\theta}{\sin\theta}\left(\frac{1+\cos\theta}{1+\cos\theta}\right)\\ &= \frac{1-\cos^2\theta}{\sin\theta+\sin\theta\cos\theta}\\ &= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} && \text{(Pythagorean identity)}\\ &= \frac{\sin\theta}{1+\cos\theta}.\end{aligned}$$

**28.** Starting from the right side, we have

$$\begin{aligned} 2\sin x\cos x\sec(2x) &= \sin(2x)\sec(2x) && \text{(Double-Angle formula)} \\ &= \tan(2x). \end{aligned}$$

**30.** Working on the left side, we have

$$\begin{aligned} \tan\left(\frac{x}{2}\right)(1+\cos x) &= \left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)(1+\cos x) && \text{(Half-Angle formula)} \\ &= \sqrt{\frac{(1+\cos x)^2(1-\cos x)}{1+\cos x}} \\ &= \sqrt{1-\cos^2 x} \\ &= \sqrt{\sin^2(x)} && \text{(Pythagorean identity)} \\ &= \sin x. \end{aligned}$$

**42.**

$$\begin{aligned} \tan\left(\frac{\pi}{12}\right) &= \sqrt{\frac{1-\cos\left(\frac{\pi}{6}\right)}{1+\cos\left(\frac{\pi}{6}\right)}} \\ &= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} \\ &= \sqrt{7-4\sqrt{3}} \\ \text{or } &= 2-\sqrt{3}. \end{aligned}$$

**44.**

$$\sin(7x)\cos(3x) = \frac{1}{2}\left[\sin(10x) - \sin(4x)\right].$$

**46.**

$$\begin{aligned} \cos(75^\circ) + \cos(15^\circ) &= 2\cos(45^\circ)\cos(30^\circ) \\ &= \sqrt{6}2. \end{aligned}$$

**48.** Working on the left side, we have

$$\begin{aligned} \frac{\sin(2x)+\sin(6x)}{\sin(2x)-\sin(6x)} &= \frac{2\sin(4x)\cos(-2x)}{2\sin(-2x)\cos(4x)} \\ &= -\frac{\sin(4x)\cos(2x)}{\cos(4x)\sin(2x)} \\ &= -\tan(4x)\cot(2x). \end{aligned}$$

# Chapter 6 Review Exercises

**2.** $A = 43°$, $B = 107°$, $C = 30°$, $a = 171.86$, $b = 240.99$, $c = 126$

**4.** $A = 54.70°$, $B = 27.41°$, $97.89°$, $a = 117$, $b = 66$, $c = 142$

**6.** Triangle 1: $A = 39°$, $B = 54.9°$, $C = 86.1°$, $a = 20$, $b = 26$, $c = 31.71$
Triangle 2: $A = 39°$, $B = 125.01°$, $C = 15.9°$, $a = 20$, $b = 26$, $c = 8.7$

**8.** $A = 162°$, $B = 3.37°$, $C = 14.63°$, $a = 58.95$, $b = 11.2$, $c = 48.2$

**12.** $A = 23°$, $B = 9.15°$, $C = 147.85°$, $a = 54.3$, $b = 22.1$, $c = 73.95$

**14.** $\text{Area} = \frac{1}{2}(4\,\text{ft})(5\,\text{ft})\sin(22°) \approx 3.75\,\text{ft}^2$

**16.** $s = \frac{1}{2}(2\,\text{m} + 2\,\text{m} + 2\,\text{m}) = 3\,\text{m}$, so by Heron's formula
$\text{Area} = \sqrt{s(s - 2\,\text{m})(s - 2\,\text{m})(s - 2\,\text{m})} = \sqrt{3\,\text{m}^4} \approx 1.73\,\text{m}^2$.

**18.** $35.6\,\text{mi}$