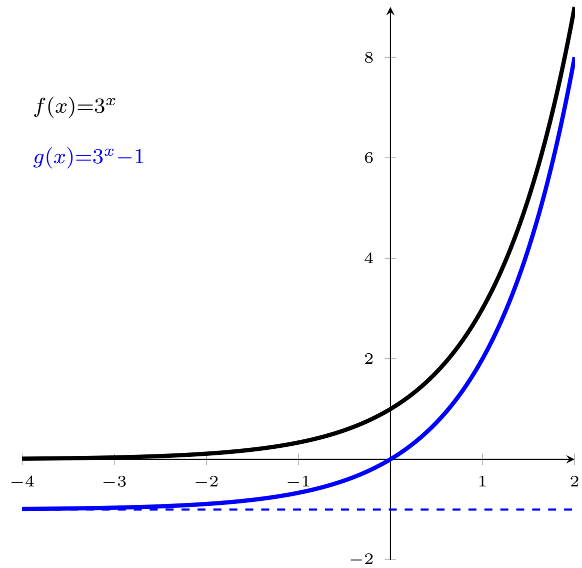
Chapter 3 Review Exercises

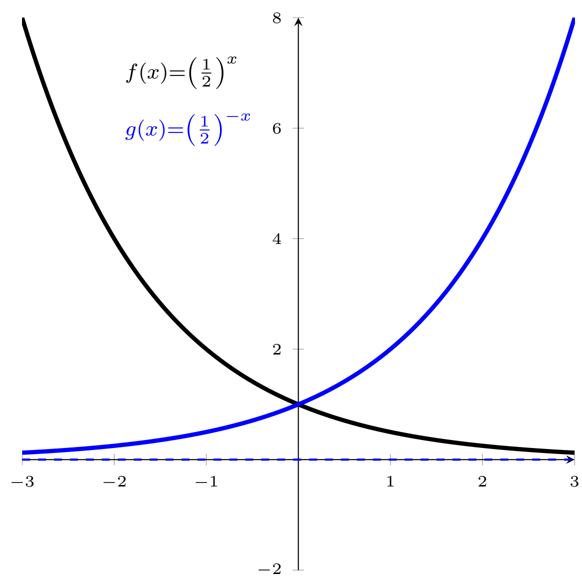
2. $h(x) = -4x^{-x}$

4. $f(x) = 4^x$

6. Horizontal asymptote for f is $y = 0$. Horizontal asymptote for g is $y = -1$.



8. Horizontal asymptote for f is $y = 0$. Horizontal asymptote for g is $y = 0$.



10. Compounded Semiannually: $\$5000\left(1+\frac{0.055}{2}\right)^{2(5)} = \$6558.26$.

Compounded Monthly: $\$5000\left(1+\frac{0.0525}{12}\right)^{12(5)} = \$6497.16$.

So, after 5 years, you make more with the account returning 5.5%, compounded semiannually.

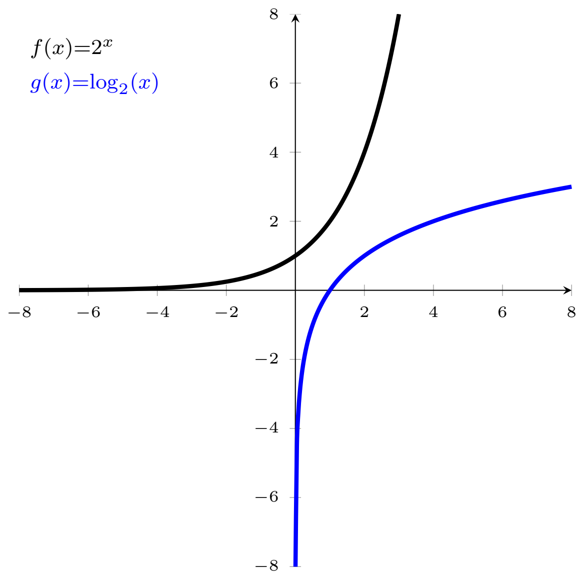
14. $4^3 = x$

16. $\log_6(216) = 3$

20. $\log_5\left(\frac{1}{25}\right) = \log_5(5^{-2}) = -2$

22. $\log_{16}(4) = \log_{16}\left(16^{1/2}\right) = \frac{1}{2}$

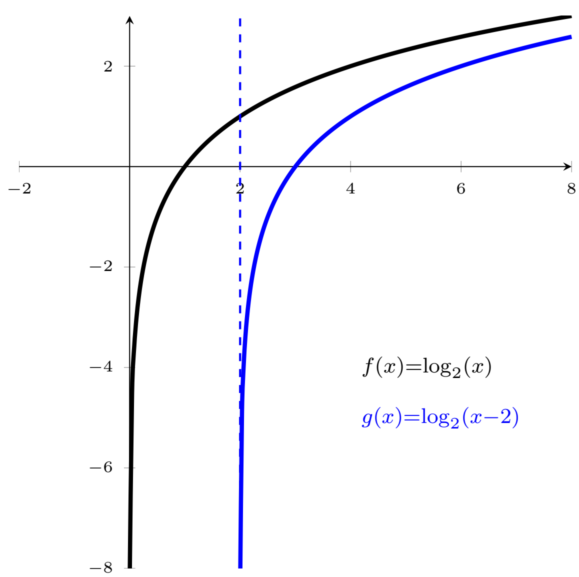
30. Domain for f is $(-\infty, \infty)$ and the range for f is $(0, \infty)$.
Domain for g is $(0, \infty)$ and the range for f is $(-\infty, \infty)$.



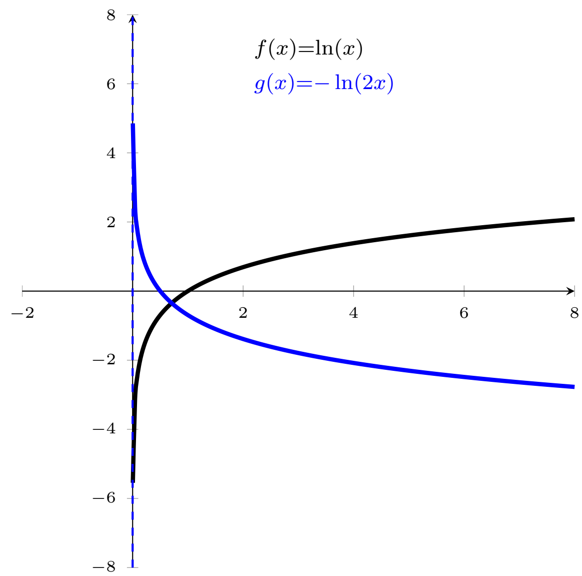
32. $g(x) = \log(-x)$

34. $h(x) = \log(2 - x)$

36. f has an x-intercept at $x = 1$, a vertical asymptote at $x = 0$, domain $(0, \infty)$, and range $(-\infty, \infty)$.
g has an x-intercept at $x = 3$, a vertical asymptote at $x = 2$, domain $(2, \infty)$, and range $(-\infty, \infty)$.



40. f has a vertical asymptote at $x = 0$, domain $(0, \infty)$, and range $(-\infty, \infty)$.
g has a vertical asymptote at $x = 0$, domain $(2, \infty)$, and range $(-\infty, \infty)$.



42. The domain is all x where $3 - x > 0$. Solving this equation yields $x < 3$, hence the domain is $(-\infty, 3)$.

44. $\ln(e^{6x}) = 6x$

50. $\log_6(36x^3) = \log_6(36) + 3\log_6(x) = \log_6(6^2) + 3\log_6(x) = 2 + 3\log_6(x)$

52. $\log_2\left(\frac{xy^2}{64}\right) = \log_2(x) + 2\log_2(y) - \log_2(64) = \log_2(x) + 2\log_2(y) - 6$

54. $\log_b(7) + \log_b(3) = \log_b(7 \cdot 3) = \log_b(21)$

56. $3\ln(x) + 4\ln(y) = \ln(x^3y^4)$

58. $\log_6(72\,348) = \frac{\ln(72\,348)}{\ln(6)} \approx 6.24$

60. Since $\ln(1) = 0$, $(\ln(x))(\ln(1)) = (\ln(x))(0) = 0$, so the equation is true.

64. $2^{4x-2} = 64 = 2^6$, so $4x - 2 = 6$, whence $x = 2$.

66. $10^x = 7000 = 7 \cdot 10^3$, so

$$\begin{aligned} \log(10^x) &= \log(7000) \\ x &= \log(7) + \log(10^3) \\ x &= \log(7) + 3 \\ x &\approx 3.85. \end{aligned}$$

72. $3^{x+4} = 7^{2x-1}$, so

$$\begin{aligned}
\ln\left(3^{x+4}\right) &= \ln\left(7^{2x-1}\right) \\
(x+4)\ln(3) &= (2x-1)\ln(7) \\
x\ln(3) + 4\ln(3) &= 2x\ln(7) - \ln(7) \\
x\ln(3) - 2x\ln(7) &= -\ln(7) - 4\ln(3) \\
x\left(\ln(3) - 2\ln(7)\right) &= -\ln(7) - 4\ln(3) \\
x &= \frac{-\ln(7) - 4\ln(3)}{\ln(3) - 2\ln(7)} \\
x &\approx 2.27.
\end{aligned}$$

74. $\log_4(3x-5) = 3$ can be written equivalently as $3x - 5 = 4^3 = 64$, whence $x = 23$.

76. $4 = \log_2(x+3) + \log_2(x-3) = \log_2((x+3)(x-3)) = \log_2(x^2-9)$. This can be written equivalently as $2^4 = x^2 - 9$, so we get $x = \pm 5$ as possible solutions. Now, -5 cannot be a solution because it is not in the domain of $\log_2(x+3) + \log_2(x-3)$, hence our only solution is $x = 5$.

80. We're solving the equation $4.6 = 14.7e^{-0.21x}$ for x, so

$$\begin{aligned}
\frac{4.6}{14.7} &= e^{-0.21x} \\
\ln\left(\tfrac{4.6}{14.7}\right) &= -0.21x \\
x &= \frac{\ln\left(\frac{4.6}{14.7}\right)}{-0.21} \\
x &\approx 5.5\,\text{mi}.
\end{aligned}$$

84. The equation representing the account balance after t years is $A(t) = \$50\,000e^{0.075t}$. We're solving the equation $\$150\,000 = \$50\,000e^{0.075t}$ for t, so

$$\begin{aligned}
3 &= e^{0.075t} \\
\ln(3) &= 0.075t \\
t &= \frac{\ln(3)}{0.075} \\
t &\approx 14.6\,\text{yr}.
\end{aligned}$$

86.

a. We know that 10 years after 1990, there were 35.3 million. We're solving $35.3 = 22.4e^{k(10)}$ for k, so

$$\begin{aligned}
\tfrac{35.3}{22.4} &= e^{10k} \\
\ln\left(\tfrac{35.3}{22.4}\right) &= 10k \\
k &= \frac{\ln\left(\frac{35.3}{22.4}\right)}{10} \\
k &\approx 0.045.
\end{aligned}$$

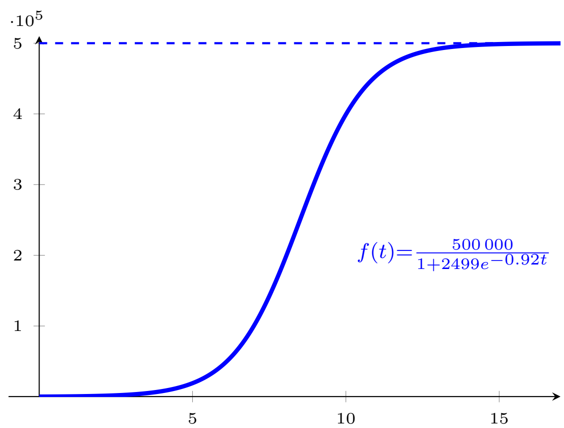
b. We now have that $A(t) = 22.4e^{0.045t}$. Since 2010 is 20 years after 1990, the population in 2010 will be $A(20) = 22.4e^{0.045(20)} \approx 55.1$ million.

c. We are solving $60 = 22.4e^{0.045t}$ for t, so

$$\begin{aligned} 60 &= 22.4e^{0.045t} \\ \frac{60}{22.4} &= e^{0.045t} \\ \ln\left(\frac{60}{22.4}\right) &= 0.045t \\ t &= \frac{\ln\left(\frac{60}{22.4}\right)}{0.045} \\ t &\approx 21.9\,\text{yr}, \end{aligned}$$

so this population will occur towards the end of 2011.

88.



a. The influenza began at time $t = 0$, so $f(0) = \dfrac{500\,000}{1 + 2499e^{-0.92(0)}} = 200$ people became ill with the flu when the epidemic began.

b. At 6 weeks after the epidemic began, $f(6) = \dfrac{500\,000}{1 + 2499e^{-0.92(6)}} \approx 45\,411$ people were ill with the flu.

c. The limiting behavior is the horizontal asymptote as $t \to \infty$. We notice that as $t \to \infty$, $e^{-0.92t} \to 0$, so $f(t) \to 500\,000$. Indeed, the horizontal asymptote, and thus the limiting size of the population that becomes ill is 500 000 people.

92. $y = 73(2.6)^x = 73e^{x\ln(2.6)}$. Since $\ln(2.6) \approx 0.956$, we have that $y \approx 73e^{0.956x}$.

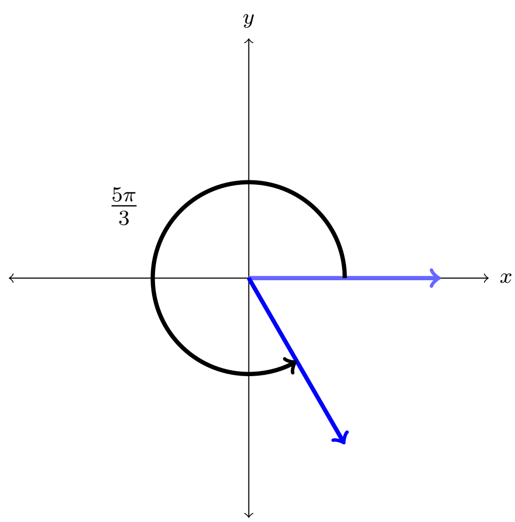
Chapter 4 Review Exercises

2. $15^\circ \left(\dfrac{\pi \,\text{rad}}{180^\circ}\right) = \frac{5\pi}{12}\,\text{rad}$

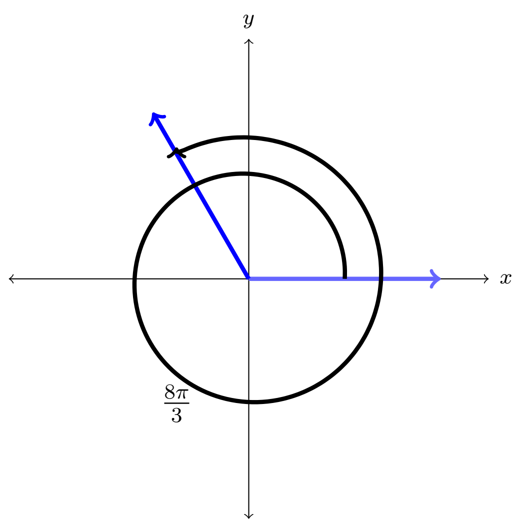
4. $315^\circ \left(\dfrac{\pi \,\text{rad}}{180^\circ}\right) = \frac{7\pi}{4}\,\text{rad}$

6. $\dfrac{7\pi}{5} \left(\dfrac{180^\circ}{\pi \,\text{rad}}\right) = 252^\circ$

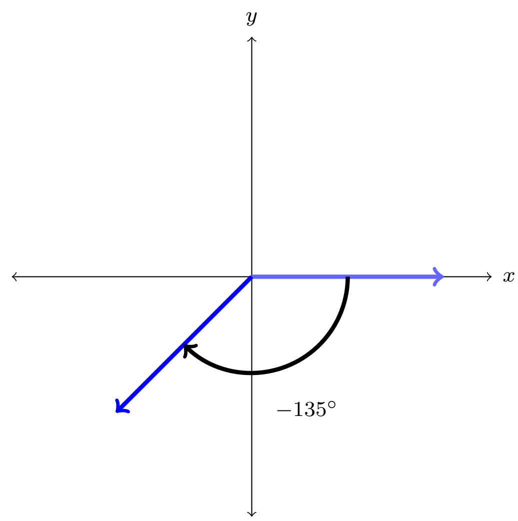
8.



10.



12.



14. $-445° + 2(360°) = 275°$

16. $\frac{31\pi}{6} - 2(2\pi) = \frac{7\pi}{6}$