Section 5.1

Fundamental Trig Identities

- **Reciprocal Identities**

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

- **Quotient Identities**

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

- **Pythagorean Identities**

$\cos^2\theta + \sin^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad \cot^2\theta + 1 = \csc^2\theta$

- **Even/Odd Identities**

$\sin(-\theta) = -\sin(\theta)$ $\quad \cos(-\theta) = \cos(\theta)$ $\quad \tan(-\theta) = -\tan(\theta)$

$\csc(-\theta) = -\csc(\theta)$ $\quad \sec(-\theta) = \sec(\theta)$ $\quad \cot(-\theta) = -\cot(\theta)$

Techniques for Verifying Trig Identities:

- Work with each side independently
- Analyze the identity and look for ways to apply fundamental identities.
- Rewrite in terms of sines & cosines
- Factor out the greatest common factor
- Separate or Combine Fractions, $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.
- Rewrite Fractions w/ least common denominator
- Don't be afraid to stop and start over.

Ex $\cos\theta \tan\theta \csc\theta = \left(\frac{\cos\theta}{1}\right)\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$ (quotient/reciprocal idents)

$= 1$

Ex $\cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta)$ (Pythag $\sin^2+\cos^2=1$)

$= \cos^2\theta - 1 + \cos^2\theta$

$= 2\cos^2\theta - 1$

Ex $\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta$

$= \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$ (PYTHAGOREAN IDENTITY)

$= \left(\frac{\cos\theta}{\sin\theta}\right)\cos\theta$

$= \cot\theta\cos\theta$ (QUOTIENT IDENTITY)

Ex $\frac{1+\cos t}{1-\cos t} = \frac{1+\cos t}{1-\cos t}\left(\frac{1+\cos t}{1+\cos t}\right)$

$= \frac{(1+\cos t)^2}{1-\cos^2 t}$

$= \frac{(1+\cos t)^2}{\sin^2 t}$ (PYTHAGOREAN IDENTITY)

$= \left(\frac{1+\cos t}{\sin t}\right)^2$

$= \left(\frac{1}{\sin t} + \frac{\cos t}{\sin t}\right)^2$

$= (\csc t + \cot t)^2$ (RECIPROCAL / QUOTIENT IDENTITIES)

Ex $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$ (PYTHAGOREAN IDENT)

Ex $\frac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x/\cos^2 x}$ (QUOTIENT IDENT)

$= \frac{\cos^2 x}{\cos^2 x}\left(\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x/\cos^2 x}\right)$

$= \cos^2 x\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x}\right)$

$= \cos^2 x$