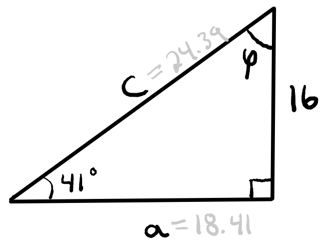
Section 4.8

Solving Right Triangles:

Given any two side lengths OR a side length and an angle in a right triangle, we can solve for the remaining information.

Ex



Find a, c, φ.

Notice that $\tan(41°) = \frac{16}{a}$, so $a = \frac{16}{\tan(41°)} \approx 18.41$

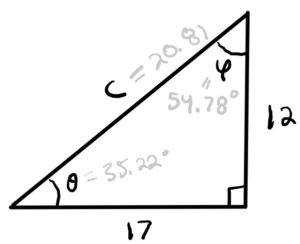
Now that we have a, we can solve for c in a few different ways:

$\cos(41°) = \frac{18.41}{c}$, $c = \sqrt{18.41^2 + 16^2}$. In either case, we get $c \approx 24.39$.

To find φ, we have even more options:

$\sin(\varphi) = \frac{18.41}{24.39}$, $\cos(\varphi) = \frac{16}{24.39}$, $\tan(\varphi) = \frac{18.41}{16}$, or even avoiding inverse trig and remembering $41° + \varphi = 90°$. We get $\varphi = 49°$.

Ex



Find c, θ, φ.

By the Pythagorean theorem, $c = \sqrt{12^2 + 17^2} \approx 20.81$

Now, $\tan(\theta) = \frac{12}{17}$, or $\cot(\theta) = \frac{17}{12}$, or $\sin(\theta) = \frac{12}{20.81}$, or $\cos(\theta) = \frac{17}{20.81}$. Using $\theta = \arctan\left(\frac{12}{17}\right) \approx 35.22°$.

Similarly, there are a million-and-five ways to get φ. Choosing $\sin(\varphi) = \frac{17}{20.81}$, we have $\varphi = \arcsin\left(\frac{17}{20.81}\right) \approx 54.78°$

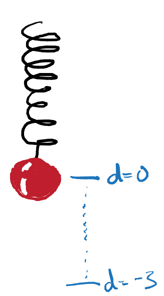
The important thing when solving a triangle is that you need at least one side length. (If you recall from geometry, there are many triangles w/ same internal angles, but different side lengths, hence AAA similarity and not AAA congruence.)

One important application of trig comes in the study of simple harmonic motion in physics.

The canonical example of simple harmonic motion is a mass on a spring. (see animations.physics.unsw.edu.au/jw/SHM.htm).
The mass has an equilibrium position with the mass attached.
If we pull the mass and stretch the spring. Ignoring things like friction and air resistance, the mass would stay bobbing up and down forever.

Def An object that moves on a coordinate axis is in **simple harmonic motion** if its distance, d, from the equilibrium position at time t is modeled by $d = a\cos(\omega t)$ or $d = a\sin(\omega t)$.
This motion has **amplitude** $|a|$. The **period** is given by $\frac{2\pi}{\omega}$, where $\omega > 0$.

Ex



Suppose a ball is attached to a spring and pulled down 3 feet from its equilibrium position. When released, it takes 5 seconds to bob up and come back down to the release point.
The period is $\frac{2\pi}{\omega} = 5 \Rightarrow \omega = \frac{2\pi}{5}$. Since the ball started down, we have that $a = -3$. So the eq. modeling the motion of the ball is $d = -3\cos\left(\frac{2\pi}{5}t\right)$.

THE FREQUENCY OF THE BALL'S MOTION WAS $\frac{1}{5}$th OF A CYCLE PER SECOND.

DEF AN OBJECT IN SIMPLE HARMONIC MOTION GIVEN BY $d = a\sin(\omega t)$ OR $d = a\cos(\omega t)$ HAS FREQUENCY $f = \frac{\omega}{2\pi} = \frac{1}{\text{period}}$, $\omega > 0$.

Ex AN OBJECT IS IN SIMPLE HARMONIC MOTION GIVEN BY $d = 14\cos\left(\frac{\pi}{4}t\right)$. THE FREQUENCY IS $f = \frac{\omega}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8}$, SO THE PERIOD IS 8 SECONDS.