SECTION 4.8

ON THE DOMAIN $(-\infty, \infty)$, WE SEE THAT $f(x) = \sin(x)$ IS VERY MUCH NOT ONE-TO-ONE, AND THUS NOT INVERTIBLE. HOWEVER, RESTRICTING THE DOMAIN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, IT IS VERY MUCH INVERTIBLE.

DEF THE INVERSE SINE FUNCTION, DENOTED $\sin^{-1}$ OR $\arcsin$, IS THE INVERSE OF $y = \sin(x)$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. THUS $y = \sin^{-1} x$ IS EQUIVALENT TO $x = \sin(y)$.

$y = \sin^{-1}(x) \neq \frac{1}{\sin(x)}$. THE -1 EXPONENT IS MEANT TO DENOTE THE INVERSE FUNCTION, LIKE $f^{-1}(x)$. OF COURSE, THIS CONFLICTS WITH NOTATION LIKE $\sin^2(x) = (\sin(x))^2$. FOR THIS REASON, I WILL USE $y = \arcsin(x)$ WHENEVER POSSIBLE TO AVOID CONFLICTING NOTATION.

[GRAPH]

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Ex $\sin^{-1}\left(\frac{1}{2}\right) = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

DEF THE INVERSE COSINE FUNCTION, DENOTED $\cos^{-1}$ OR $\arccos$, IS THE INVERSE OF $y = \cos(x)$, $0 \le x \le \pi$. $y = \cos^{-1}(x)$ IS EQUIVALENT TO $x = \cos(y)$.

[GRAPH]

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

Ex

Def The **inverse tangent function**, denoted $\tan^{-1}$ or $\arctan$, is the inverse of $y = \tan(x)$, $\frac{\pi}{2} < x < \frac{\pi}{2}$. $y = \tan^{-1}(x)$ is equivalent to $\tan(y) = x$

[GRAPH]

θ	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan\theta$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Ex

Inverse Properties:

$\sin(\sin^{-1}(x)) = x$, for x in $[-1, 1]$

$\sin^{-1}(\sin(x)) = x$, for x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\cos(\cos^{-1}(x)) = x$, for x in $[-1, 1]$

$\cos^{-1}(\cos(x)) = x$, for x in $[0, \pi]$

$\tan(\tan^{-1}(x)) = x$, for x in $(-\infty, \infty)$

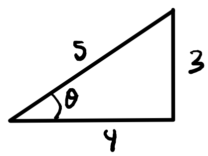
$\tan^{-1}(\tan(x)) = x$, for x in $(-\frac{\pi}{2}, \frac{\pi}{2})$

Ex $\sin^{-1}(\sin(0)) = 0$ because 0 is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$\sin^{-1}(\sin(\pi)) \neq \pi$, because π is not in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Instead, we have to actually evaluate: $\sin^{-1}(\sin(\pi)) = \sin^{-1}(0) = 0$.

Ex Find the exact value of $\sin(\tan^{-1}(\frac{3}{4}))$. $\frac{3}{4}$ is in domain.

$\tan^{-1}(\frac{3}{4}) = \theta \Leftrightarrow \tan(\theta) = \frac{3}{4}$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, so $\theta > 0$.



$r = \sqrt{3^2 + 4^2} = \sqrt{25}$. Then

$\sin(\tan^{-1}(\frac{3}{4})) = \sin(\theta) = \frac{3}{r} = \frac{3}{5}$