Section 4.4

Let θ be any angle in standard position, and $P(x,y)$ a point on the terminal side of θ. If $r=\sqrt{x^2+y^2}$ is the distance from $(0,0)$ to (x,y), the six trig functions are given by

$$\sin(\theta)=\frac{y}{r} \qquad \csc(\theta)=\frac{r}{y},\ y\neq 0$$

$$\cos(\theta)=\frac{x}{r} \qquad \sec(\theta)=\frac{r}{x},\ x\neq 0$$

$$\tan(\theta)=\frac{x}{y},\ y\neq 0 \qquad \cot(\theta)=\frac{y}{x},\ x\neq 0$$

Ex Let $P(-7,1)$ be on the terminal side of θ. Then

$$r=\sqrt{(-7)^2+1^2}=\sqrt{50}=5\sqrt{2}$$

$$\sin(\theta)=\frac{1}{5\sqrt{2}}=\frac{\sqrt{2}}{10} \qquad \csc(\theta)=5\sqrt{2}$$

$$\cos(\theta)=-\frac{7}{5\sqrt{2}}=\frac{-7\sqrt{2}}{10} \qquad \sec(\theta)=-\frac{5\sqrt{2}}{7}$$

$$\tan(\theta)=-\frac{1}{7} \qquad \tan(\theta)=-7$$

One thing we notice is that the sign of the trig functions depend on the quadrant.

QII	QI
$\sin(\theta)>0$	$\sin(\theta)>0$
$\cos(\theta)<0$	$\cos(\theta)>0$
QIII	**QIV**
$\sin(\theta)<0$	$\sin(\theta)<0$
$\cos(\theta)<0$	$\cos(\theta)>0$

So in fact, we only need one trig function and info about the quadrant to determine the remaining trig functions.

Ex $\tan(\theta)=-\frac{1}{3}$, $\sin(\theta)>0$.

Since $\sin(\theta)>0$ and $\tan(\theta)<0$, $\cos(\theta)<0$.

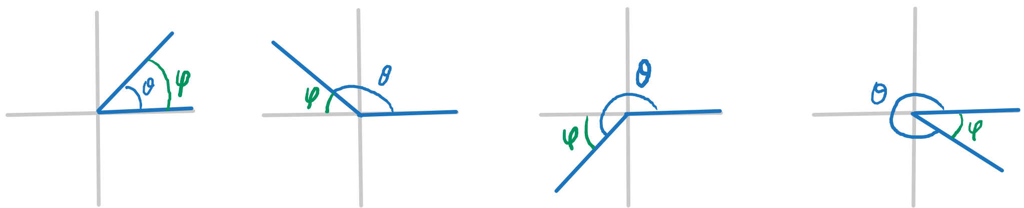
$r=\sqrt{1^2+(-3)^2}=\sqrt{10}$, so

$$\sin(\theta)=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10} \qquad \csc(\theta)=\sqrt{10} \qquad \cot=-3.$$

$$\cos(\theta)=-\frac{3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10} \qquad \sec(\theta)=-\frac{\sqrt{10}}{3}$$

Def Let θ be a nonacute (ie, $|\theta| > 90^\circ$ or $\frac{\pi}{2}$) angle. The **reference angle** is the angle φ formed by the terminal side of θ and the x-axis.

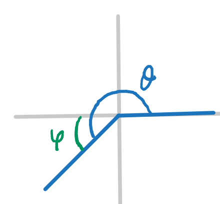
Ex



Ex $\theta = 31^\circ$, $\varphi = 31^\circ$
$\theta = 170^\circ$, $\varphi = 10^\circ$
$\theta = -170^\circ$, $\varphi = 10^\circ$
$\theta = \frac{13\pi}{6}$, $\varphi = \frac{\pi}{6}$

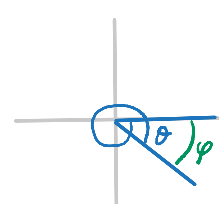
Using Reference Angles to Find Trig Values

Ex $\theta = \frac{5\pi}{4}$, so $\varphi = \frac{\pi}{4}$. Now, $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, and since θ is in QIII, $\sin(\theta) < 0$. Hence $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.



The whole point of these reference angles is that it allows us to think about only one quadrant of the unit circle, and know that every other quadrant results in trig values that are the same up to a sign.

Ex $\theta = -\frac{7\pi}{3}$, so $\varphi = \frac{\pi}{3}$. Now, $\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{1/2} = 2$ and θ is in QIV, so $\cos\theta > 0$ $\Rightarrow \sec\theta > 0$. Hence $\sec\left(-\frac{7\pi}{3}\right) = 2$.

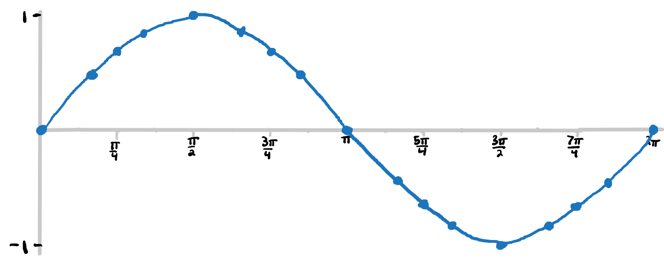


Section 4.5

Graphing $y=\sin(x)$

x	$\sin(x)$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$

x	$\sin(x)$
π	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{11\pi}{6}$	$-\frac{1}{2}$



Graph of $y=\sin(x)$

Above we see, graphically, what sine looks like. Recall that sine had a period 2π. What this means is, if we continued graphing this, we would see the same shape repeated over and over.

One thing we notice is that the graph of $\sin(x)$ reaches its maximum at $\frac{1}{4}$ of the period, it's minimum at $\frac{3}{4}$ of the period, and has x-intercepts at the beginning, middle and end of the period.

Let's look at some transformations of $y=\sin(x)$.

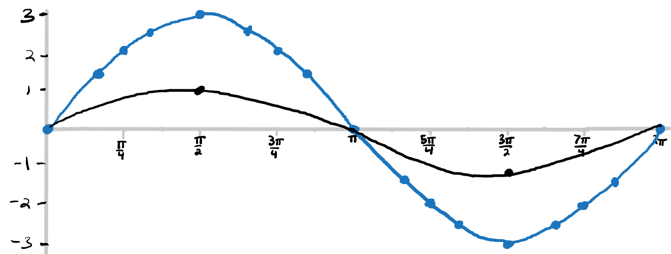
Amplitude

Def Let $y=A\sin(x)$ for $A\neq 0$. Then $|A|$ is called the **amplitude** of $y=A\sin(x)$. This function has range $[-|A|,|A|]$.

Ex $y=3\sin(x)$ IS A VERTICAL STRETCH OF $\sin(x)$, AS EXPECTED.

x	$3\sin(x)$
0	0
$\frac{\pi}{6}$	$\frac{3}{2}$
$\frac{\pi}{4}$	$\frac{3\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{3\sqrt{3}}{2}$
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	$\frac{3\sqrt{3}}{2}$
$\frac{3\pi}{4}$	$\frac{3\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{3}{2}$

x	$\sin(x)$
π	0
$\frac{7\pi}{6}$	$-\frac{3}{2}$
$\frac{5\pi}{4}$	$-\frac{3\sqrt{2}}{2}$
$\frac{4\pi}{3}$	$-\frac{3\sqrt{3}}{2}$
$\frac{3\pi}{2}$	-3
$\frac{5\pi}{3}$	$-\frac{3\sqrt{3}}{2}$
$\frac{7\pi}{4}$	$-\frac{3\sqrt{2}}{2}$
$\frac{11\pi}{6}$	$-\frac{3}{2}$



GRAPH OF $y=\sin(x)$

GRAPH OF $y=3\sin(x)$

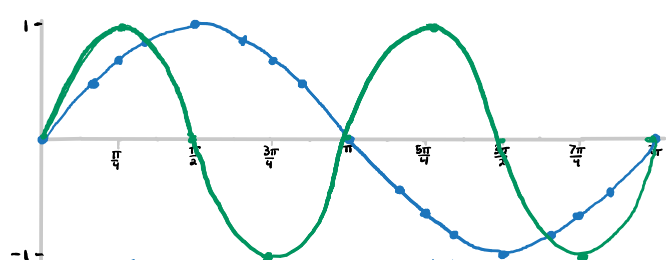
NOTICE THAT $y=A\sin(x)$ AND $y=\sin(x)$ HAVE THE SAME PERIOD, AND SAME LOCATIONS OF MAXIMA, MINIMA, AND X-INTERCEPTS.

PERIOD

Ex $y=\sin(2x)$ IS A HORIZONTAL SHRINK, AS EXPECTED.

x	$\sin(2x)$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{3\pi}{4}$	-1
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$

x	$\sin(2x)$
π	0
$\frac{7\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{4}$	1
$\frac{4\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{7\pi}{4}$	-1
$\frac{11\pi}{6}$	$-\frac{\sqrt{3}}{2}$



GRAPH OF $y=\sin(x)$

GRAPH OF $y=\sin(2x)$

$y=A\sin(Bx)$ HAS PERIOD $\frac{2\pi}{B}$.