Section 3.3 (Continued)

Using properties of logarithms, we can also condense expressions.

Ex $\ln(18x-2) - \ln(y) - \ln(41) = \ln\left(\frac{18x-2}{41y}\right)$ (Use PEMDAS.)

Ex $\log_2(3) - \frac{1}{2}\log_2(x+15) = \log_2\left(\frac{3}{\sqrt{x+15}}\right)$

We have one more property that will prove extremely useful when explicitly computing logarithms. First, let $a, b > 0$ with $a, b \neq 1$, and let $M > 0$. Set $x = \log_a(M)$ and $y = \log_b(M)$, so that $M = a^x = b^y$. Then

$$\begin{aligned} y = \log_b(M) &= \log_b(a^x) \\ &= x \log_b(a) \\ &= \log_a(M)\log_b(a). \end{aligned}$$

$$\Rightarrow \log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

Change of Base Property for Logarithms. Let $a, b > 0$, w/ $a, b \neq 1$, and $M > 0$. Then $\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$.

In particular, for our calculators, this means

$$\log_a(M) = \frac{\log(M)}{\log(a)} = \frac{\ln(M)}{\ln(a)}.$$

Ex $\log_7(82) = \frac{\ln(82)}{\ln(7)} \approx 2.26461$

Section 3.4

Suppose we want to solve for x in $3^{x+1} = 81$.
How do we do it?

Expressing Each Side As a Power of the Same Base

1. Rewrite equation in the form of $b^M = b^N$.
2. Set $M = N$.
3. Solve.

Ex $3^{x+1} = 81 = 3^4 \Rightarrow x+1 = 4 \Rightarrow x = 3$.

Use Logarithms to Solve Exponential Equations

1. Isolate Exponential Expression
2. Take a logarithm (w/ same base) of both sides.
3. Simplify w/ power rule for logarithms.
4. Solve.

Ex $3^{x+1} = 81 \Rightarrow \ln(3^{x+1}) = \ln(81)$

$\Rightarrow (x+1)\ln(3) = \ln(81)$

$\Rightarrow x+1 = \frac{\ln(81)}{\ln(3)}$

$\Rightarrow x = \frac{\ln(81)}{\ln(3)} - 1$ ....simplify.... $= 4 \cdot \frac{\ln(3)}{\ln(3)} - 1 = 3$.

Use Definition of a Logarithm

1. Express as $\log_b(M) = c$.
2. Rewrite as $b^c = M$.
3. Solve.

Ex $\log_3(2x-6)=15 \Rightarrow 3^{15}=2x-6 \Rightarrow 2x=3^{15}+6 \Rightarrow x=\frac{1}{2}(3^{15})+3.$

Using 1-to-1 Property of Logs

1. Rewrite as $\log_b(M)=\log_b(N)$
2. Set $M=N$.
3. Solve.

Ex $\log(x)=\log(2)-\log(15) \Rightarrow \log(x)=\log\left(\frac{2}{15}\right) \Rightarrow x=\frac{2}{15}.$