Section 3.2

We said last time that, for $b>0$ with $b \neq 1$, the function $f(x)=b^x$ has an inverse.

Step 1: $y=b^x$

Step 2: $x=b^y$

Step 3: ?

We don't have a way to handle this. So, we create new notation:

Def For $x \neq 0$, $b>0$, and $b \neq 1$,

$y=\log_b(x)$ is equivalent to $x=b^y$.

$f(x)=\log_b(x)$ is the **logarithmic function with base b.**

Ex $3=\log_5 x \Rightarrow 5^3=x \Rightarrow x=125$

$y=\log_2 64 \Rightarrow 2^y=64 \Rightarrow y=6$

$3=\log_b 27 \Rightarrow b^3=27 \Rightarrow b=3$

Basic Logarithm Properties: Let $b>0$ and $b \neq 1$. Then

1) $\log_b(b)=1$, since $b^1=b$.
2) $\log_b(1)=0$, since $b^0=1$

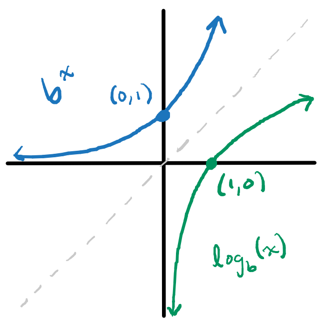
We said that, for $f(x)=b^x$, $f^{-1}(x)=\log_b(x)$. Since $f^{-1}(f(x))=x$ and $f(f^{-1}(x))=x$,

Inverse Property For $b>0$ and $b \neq 1$, we have

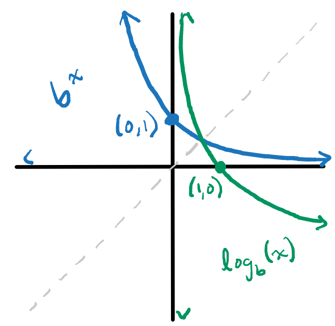
1) $f(f^{-1}(x))=f(\log_b(x))=b^{\log_b(x)}=x$, and
2) $f^{-1}(f(x))=f^{-1}(b^x)=\log_b(b^x)=x$.

Since $f(x) = b^x$ has domain $(-\infty, \infty)$ and range $(0, \infty)$, $f^{-1}(x) = \log_b(x)$ has domain $(0, \infty)$ and range $(-\infty, \infty)$.

For $b > 1$



For $0 < b < 1$



Claim All transformations of functions still hold for logarithmic functions.

Def The logarithm with base 10 is the **common logarithm**. It is often denoted as just $\log(x)$, without the base. The logarithm with base e is the **natural logarithm**. It is often denoted $\ln(x)$. This is the convention we'll adhere to in our class.

Note: WolframAlpha and other higher-level math texts use $\log(x)$ or $\mathrm{Log}(x)$ to denote the natural logarithm.

Ex The Richter scale, which measures earthquake intensity, is logarithmic. So if an earthquake is 10^k times as strong, it registers a $\log(10^k) = \log_{10}(10^k) = k$ on the Richter scale.

Ex Sound intensity in deciBels is also measured on a base-10 logarithmic scale (see exercises 117 + 118 in Section 3.2).

Section 3.3

Let $b>0$, $b\neq 1$, and let $M, N>0$ be real numbers.
Let $x=\log_b(M)$, $y=\log_b(N)$, so then $M=b^x$, $N=b^y$.

$$MN=b^x b^y=b^{x+y},$$

so then

$$\log_b(MN)=\log_b(b^{x+y})=x+y=\log_b(M)+\log_b(N).$$

Product Rule for Logarithms. For $b, M, N>0$ with $b\neq 1$, we have $\log_b(MN)=\log_b(M)+\log_b(N)$

With b, M, N, x, y as before, we have

$$M/N=b^x/b^y=b^{x-y},$$

so

$$\log_b(M/N)=\log_b(b^{x-y})=x-y=\log_b M-\log_b(N).$$

Quotient Rule for Logarithms. For $b, M, N>0$ with $b\neq 1$, we have $\log_b\left(\frac{M}{N}\right)=\log_b(M)-\log_b(N)$.

With b, M as before, and p any nonzero real number. Set $x=\log_b(M^p)$ so that $b^x=M^p$. Then $b^{x/p}=(b^x)^{1/p}=(M^p)^{1/p}=M^{p/p}=M$,

so

$$\log_b(M)=\log_b(b^{x/p})=\frac{x}{p}=\frac{1}{p}\log_b(M^p),$$
$$\Rightarrow \log_b(M^p)=p\log_b(M).$$

We only proved this for nonzero p, but the result is obviously true for $p=0$ (why?).

Power Rule for Logarithms Let $b, M>0$, $b\neq 1$, and p is any real number. Then $\log_b(M^p)=p\log_b(M)$.