Section 2.3 (Cont'd)

Def A zero (or root or solution) of a polynomial function f is is the x-value s.t. $f(x)=0$.

Finding roots is a highly nontrivial procedure, and there is no one-size-fits-all approach.

Ex 1 $f(x)=x^2-9=(x+3)(x-3)=0 \Rightarrow f$ has roots $x=\pm 3$.

$g(x)=x^3-16x-x^2+16=x(x^2-16)-(x^2-16)=(x-1)(x^2-16)=(x-1)(x-4)(x+4)=0$

$\Rightarrow g$ has roots $x=1,\pm 4$.

Remark: Real roots of a polynomial function correspond to x-intercepts on the graph.

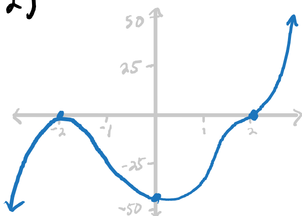
Def Let r be a root of $f(x)$. If $g(x)$ is a function such that $f(x)=(x-r)^k g(x)$ and $g(r)\neq 0$, the integer k is the multiplicity of the root r.

Ex $f(x)=x^4(x-1)^3$. 0 is a root of multiplicity 4. 1 is a root of multiplicity 3.

Def A root of multiplicity 1 is called a simple root; a root of multiplicity $k>1$ is called a multiple root.

Result If r is a root of even multiplicity touches the graph touches the x-axis at r and turns around. If it has odd multiplicity, it crosses the x-axis at r.

Ex $f(x) = (x+2)^2(x-2)^3$



Theorem (Intermediate Value) Let f be a polynomial function. If $f(a)$ and $f(b)$ have different signs (say $f(a) > 0$, $f(b) < 0$), then there exists a c s.t. $a < c < b$ and $f(c) = 0$.

Ex $f(x) = x^3 - x - 1$. Since $f(1) = -1$ and $f(2) = 5$, f has a root in the interval $(1,2)$.

Section 2.4

Theorem (Division Algorithm) Let $d(x)$, $f(x)$ be polynomials with $d(x) \neq 0$ and $\deg d(x) \leq \deg f(x)$. Then there exist unique polynomials $q(x)$ and $r(x)$ s.t. $f(x) = d(x)\,q(x) + r(x)$. If $r(x)$, the remainder is zero, we say $d(x)$ divides evenly into $f(x)$, and that $d(x)$, $q(x)$ are factors of $f(x)$.

Ex (Polynomial Long Division) $(x^4 + 2x - 1) \div (x^2 + 3x) = x^2 - 3x + 9 + \frac{-25x - 1}{x^2 + 3x}$

$$
\begin{array}{r l}
 & x^2 - 3x + 9 \\
x^2 + 3x \,\big) & x^4 + 0x^3 + 0x^2 + 2x - 1 \\
 & -(x^4 + 3x^3) \\
 & \quad -3x^3 + 0x^2 \\
 & \quad -(-3x^3 - 9x^2) \\
 & \qquad 9x^2 + 2x \\
 & \qquad -(9x^2 + 27x) \\
 & \qquad\quad -25x - 1
\end{array}
$$

Theorem (Remainder Theorem) If $f(x)$ is divided by $(x-c)$, the remainder is $f(c)$.

This allows us to solve for $f(c)$ w/o ever plugging in $f(c)$.

Ex $f(x) = 2x^3 - 11x^2 + 7x - 5$, find $f(4)$

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 5 \\
\hline
x-4 & 2x^3 - 11x^2 + 7x - 5 \\
 & -(2x^3 - 8x^2) \\
\hline
 & -3x^2 + 7x \\
 & -(-3x^2 + 12x) \\
\hline
 & -5x - 5 \\
 & -(-5x + 20) \\
\hline
 & -25 = f(4)
\end{array}
$$

Theorem (Factor Theorem) $(x-c)$ is a factor of $f(x)$ if and only if $f(c) = 0$.

Section 2.5

Theorem (Rational Root Theorem) If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has integer coefficients and p/q (in lowest terms) is a rational root of f, then p is a factor of a_0 and q is a factor of a_n.

Ex $2x^3 - 5x^2 + x + 6$

Factors of constant term: $\pm 1, \pm 2, \pm 3, \pm 6$

Factors of leading term: $\pm 1, \pm 2$

Possible rational roots: $\pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{3}{1}, \pm \frac{6}{1}, \pm \frac{1}{2}, \pm \frac{3}{2}$.