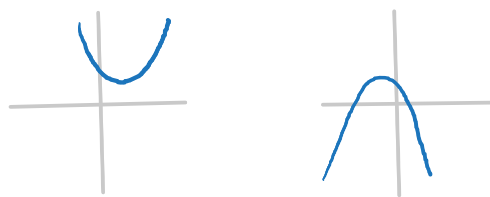
Section 2.2

Def A **quadratic function** is a function of the form $f(x)=ax^2+bx+c$, where $a \neq 0$.

If $a>0$, the parabola "opens upward". If $a<0$, it "opens downward."



$a>0$ $\quad$ $a<0$

Def The **vertex** (of the parabola) is the lowest point on the graph when it opens upward and is the highest point on the graph when it opens downward

Def The vertical line through the vertex is called the **axis of symmetry**.

Def The **standard form** for a quadratic function is $f(x)=a(x-h)^2+k$, where $a \neq 0$.

This may seem strange at first, but it's a particularly useful form as it allows us to just read off the vertex and axis of symmetry. In particular, the vertex is at the point (h,k), and the axis of symmetry is $x=h$.

Since parabolas have this nice symmetry, it means we can more easily and more accurately graph them.

To graph $f(x) = a(x-h)^2 + k$

1. Determine if it opens upward or downward.
2. Determine vertex and axis of symmetry.
3. Find any x-intercepts, solving $f(x)=0$ (real zeroes only).
4. Find any y-intercepts, $f(0)$.
5. Plot it!

Ex 1 $f(x) = 2(x+1)^2 - 2 = 2(x^2+2x+1) - 2 = 2x(x+2)$.

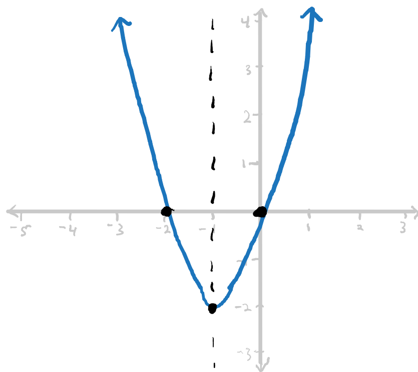
2) vertex at $(-1,-2)$, axis of symmetry is $x=-2$.

1) $2>0$, so opens up.

3) $2x(x+2) = 0 \Rightarrow x = -2, 0$ are x-intercepts.

4) $f(0) = 0$ is y-intercept.

5)



Is it possible to put every quadratic function into this standard form? Yep, via a method called "completing the square," (which is actually how you prove the quadratic formula).

Completing the Square

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x\right)+c$$
$$= a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right)+c$$
$$= a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)-\frac{b^2}{4a}+c$$
$$= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$$

Ex 2 $f(x) = 3x^2-12x+1 = 3(x^2-4x)+1$
$$= 3\left(x^2-4x+\left(\frac{4}{2}\right)^2-\left(\frac{-4}{2}\right)^2\right)+1$$
$$= 3\left(x^2-4x+(-2)^2-(-2)^2\right)+1$$
$$= 3(x^2-4x+2^2)-12+1$$
$$= 3(x-2)^2-11$$

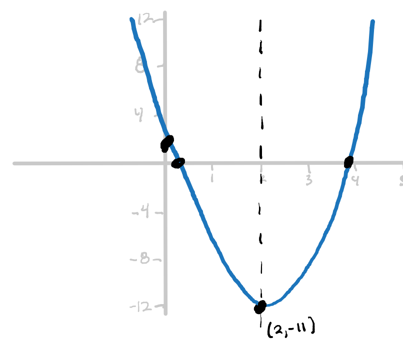
VERTEX: $(2,-11)$

AXIS OF SYMMETRY: $x=2$

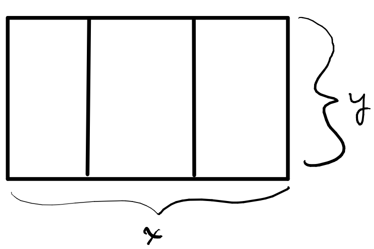
y-INTERCEPT: 1

OPENS UP

x-INTERCEPTS: $2\pm\frac{1}{3}\sqrt{33}$



Ex 3 REMEMBER OUR FARMER FROM TWO DAYS AGO? HE HAD 1200 ft OF FENCING TO BUILD THE PEN BELOW:



WE DEDUCED THAT $A(x) = -\frac{x^2}{4}+300x$ REPRESENTED THE AREA AS A FUNCTION OF SIDE LENGTH. NOW THE FARMER WANTS TO MAXIMIZE THE AREA. SINCE THE PARABOLA OPENS DOWN, THE MAX OCCURS AT THE VERTEX: $(600, 90\,000)$

$$A(x) = -\frac{x^2}{4}+300x = -\frac{1}{4}(x^2-1200x) = -\frac{1}{4}(x^2-1200x+600^2-600^2) = -\frac{1}{4}(x-600)^2+300^2$$

WHEN THE WIDTH IS 600 ft, HIS PEN ENCLOSES 90,000 ft.

SECTION 2.3

DEF A POLYNOMIAL FUNCTION IS A FUNCTION OF THE FORM $f(x) = a_n x^n + \cdots + a_1 x + a_0$, WHERE $a_0, \ldots, a_n$ ARE REAL NUMBERS. THIS IS CALLED A POLYNOMIAL OF DEGREE n AND a_n IS CALLED THE LEADING COEFFICIENT.

NOTE: FOR POLYNOMIAL FUNCTIONS, n MUST BE A NONNEGATIVE INTEGER.

Ex 4 $f(x) = x^3 + 5$, $g(x) = 3$, $h(x) = 16x - 13x^{19} + 8 - 31x^2$ ARE POLYNOMIAL FUNCTIONS.

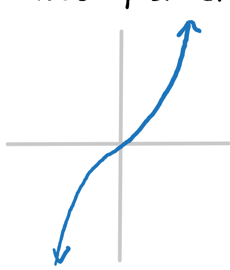
$p(x) = x^{-1} = \frac{1}{x}$, $q(x) = x^{1/2} - 3x^2 = \sqrt{x} - 3x^2$, $r(x) = 2^x$ ARE NOT POLYNOMIAL FUNCTIONS.

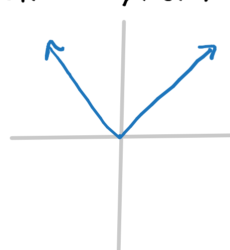
DEF A FUNCTION IS CONTINUOUS IF ITS GRAPH CAN BE DRAWN W/O LIFTING A PENCIL OFF THE PAPER. A FUNCTION IS SMOOTH IF IT HAS NO SHARP CORNERS.

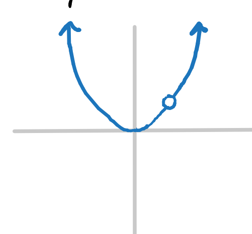
(THESE DEFINITIONS WILL WORK FOR THIS CLASS, BUT IT SHOULD BE NOTED THAT THEY ARE WAY TOO LOOSE TO BE ACTUAL MATHEMATICAL DEFINITIONS)

Ex 5 $f(x) = x^3$ $\qquad$ $g(x) = |x|$ $\qquad$ $h(x) = \frac{x^2(x-1)}{x-1}$

CONTINUOUS, SMOOTH $\qquad$ CONTINUOUS, NOT SMOOTH $\qquad$ SMOOTH, NOT CONTINUOUS







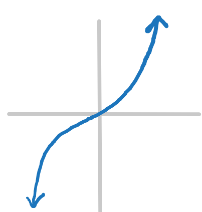
RESULT: POLYNOMIAL FUNCTIONS ARE SMOOTH AND CONTINUOUS.

Def The **end behavior** of the graph of a function is a description of what happens to the far left and far right of the graph (secretly, as x heads off to $-\infty$ and ∞, resp.)

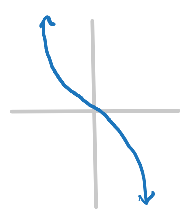
Theorem (Leading Coefficient Test) Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$, $a_n \neq 0$, be a polynomial function. We have four cases:

1) If n is odd and $a_n > 0$, the function falls to the left and rises to the right $(\swarrow \nearrow)$
2) If n is odd and $a_n < 0$, the function rises to the left and falls to the right $(\nwarrow \searrow)$.
3) If n is even and $a_n > 0$, the function rises to both the left and right $(\nwarrow \nearrow)$
4) If n is even and $a_n < 0$, the function falls to both the left and right $(\swarrow \searrow)$

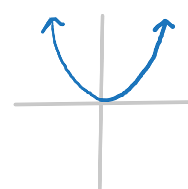
Ex 6 $f(x) = x^3$



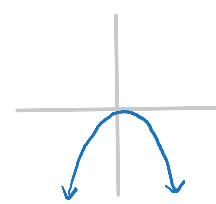
$f(x) = -x^3$



$f(x) = x^2$



$f(x) = -x^2$



Ex 7 $f(x) = -31x^{7962186351} + x^2 + 385{,}000{,}000{,}002$

By the leading coefficient test, this rises to the left and falls to the right.