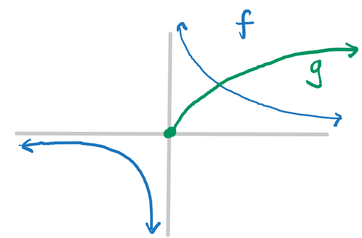
Section 1.7

BEFORE WE SAW THE NOTION OF THE DOMAIN OF A RELATION AND DETERMINED THE DOMAIN BASED ON GRAPHS. SINCE A FUNCTION IS JUST A SPECIAL TYPE OF RELATION,

DEF THE **DOMAIN** OF A FUNCTION $f(x)$ IS THE SET OF ALL x-VALUES WHERE $f(x)$ IS A REAL NUMBER.

Ex 1 $f(x) = \frac{1}{x}$ HAS DOMAIN $(-\infty, 0) \cup (0, \infty)$.
$g(x) = \sqrt{x}$ HAS DOMAIN $[0, \infty)$.



LET f, g BE TWO FUNCTIONS W/ DOMAINS D_f, D_g, RESPECTIVELY. LET $D = D_f \cap D_g$ BE THE SET OF ALL ELEMENTS COMMON TO BOTH D_f AND D_g. THE **SUM** $f+g$, **DIFFERENCE** $f-g$, **PRODUCT** fg, AND **QUOTIENT** f/g ARE FUNCTIONS WITH DOMAINS D DEFINED AS FOLLOWS:

1. SUM $\quad (f+g)(x) = f(x) + g(x)$
2. DIFFERENCE $\quad (f-g)(x) = f(x) - g(x)$
3. PRODUCT $\quad (fg)(x) = f(x)g(x)$
4. QUOTIENT $\quad (f/g)(x) = f(x)/g(x)$, PROVIDED $g(x) \neq 0$, SO IN FACT f/g MAY HAVE MORE RESTRICTIONS ON ITS DOMAIN.

Ex 2 LET $f(x) = x+1$, $g(x) = x^2 - 4$. BOTH HAVE DOMAINS $(-\infty, \infty)$. THEN $(f+g)(x) = x^2 + x - 3$. $(f-g)(x) = -x^2 + x + 5$. $(fg)(x) = (x+1)(x^2-4) = x^3 + x^2 - 4x - 4$. $(f/g)(x) = \frac{x+1}{x^2-4}$. THE FIRST 3 HAVE DOMAIN $(-\infty, \infty)$. THE QUOTIENT HAS DOMAIN $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

NOTE: DETERMINE THE DOMAIN <u>BEFORE</u> SIMPLIFYING.

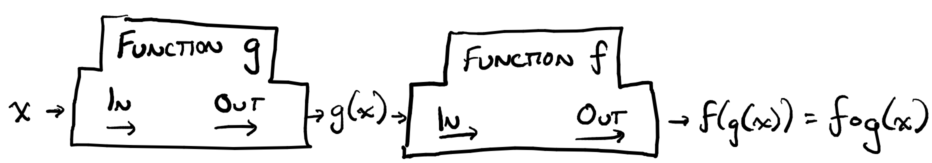
Ex 3 LET $f(x) = x^2 - 1 = (x+1)(x-1)$, $g(x) = x - 1$.

$\left(f/g\right)(x) = \dfrac{x^2-1}{x-1}$, SO f/g HAS DOMAIN $(-\infty, 1) \cup (1, \infty)$, BUT

$= \dfrac{(x+1)(x-1)}{x-1} = x + 1.$

THE **COMPOSITION OF THE FUNCTION f WITH g** IS THE FUNCTION $f \circ g$ (READ "f COMPOSED WITH g") DEFINED BY $(f \circ g)(x) = f(g(x))$. THE **DOMAIN OF $f \circ g$** IS THE SET OF ALL x SUCH THAT

1. x IS IN THE DOMAIN OF g, AND
2. $g(x)$ IS IN THE DOMAIN OF f.



Ex 4 LET $f(x) = x^2 + 1$, $g(x) = x + 1$. THEN

$$f \circ g(x) = f(g(x)) = f(x+1) = (x+1)^2 + 1 = x^2 + 2x + 2$$

$$g \circ f(x) = g(f(x)) = g(x^2+1) = (x^2+1) + 1 = x^2 + 2$$

NOTE: $f \circ g$ AND $g \circ f$ NEED NOT BE THE SAME FUNCTION!

Ex 5 LET $f(x) = x + 1$, $g(x) = \sqrt{x}$. THEN

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = \sqrt{x} + 1$$

$$g \circ f(x) = g(f(x)) = g(x+1) = \sqrt{x+1}$$

DOMAIN: $[0, \infty)$ DOMAIN: $[-1, \infty)$.

AS WE SAW WITH TRANSFORMATIONS, IT IS USEFUL TO THINK OF FUNCTIONS AS COMPOSITIONS OF SIMPLER FUNCTIONS, WE CALL THIS DECOMPOSITION.

Ex LET $h(x) = (\sqrt{x} + 7)^9$. WE CAN WRITE $h(x) = f \circ g(x)$, WHERE $f(x) = x^9$ AND $g(x) = \sqrt{x} + 7$. WE CAN ALSO CHOOSE $f(x) = (x+1)^7$ AND $g(x) = \sqrt{x}$.

DECOMPOSITIONS ARE NOT NECESSARILY UNIQUE.