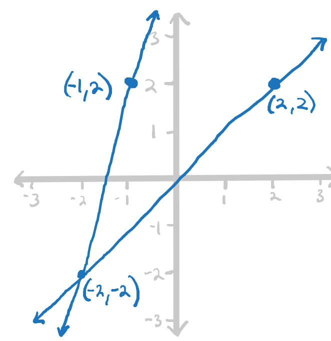
Section 1.4 (Continued)

Ex 5 The slope of the line through $(-2,-2)$ and $(2,2)$ is $\frac{(2)-(-2)}{(2)-(-2)} = \frac{2+2}{2+2} = \frac{4}{4} = 1$.

The slope of the line through $(-2,-2)$ and $(-1,2)$ is $\frac{(2)-(-2)}{(-1)-(-2)} = \frac{2+2}{-1+2} = \frac{4}{1} = 4$.



Def The point-slope form of the equation of the line with slope m through point (x_1, y_1) is $(y-y_1) = m(x-x_1)$.

Def The slope-intercept form of the equation of the line with slope m with y-intercept $(0,b)$ is $y = mx+b$.

Ex 6 The line passing through $(1,-2)$ with slope -3 has point-slope form $(y+2) = -3(x-1)$.

The line with y-intercept $(0,1)$ with slope -3 has slope-intercept form $y = -3x+1$.

Def The equation of a horizontal line through $(0,b)$ is $y=b$.
The equation of a vertical line through $(a,0)$ is $x=a$.

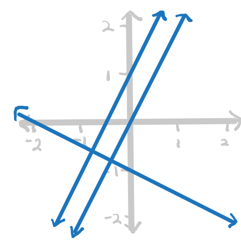
Def Every line has an equation that can be written in the general form $Ax+By+C=0$, where A, B, C are real numbers, and A and B are not both zero.

Section 1.5

Def Two lines in the same plane are **parallel** if they don't intersect. A handy feature about parallel lines is that they have the same slope.

Def Two lines are **perpendicular** if they intersect at a right ($90°$) angle. A handy feature of perpendicular lines is that their slopes are negative reciprocals of one another.

Ex 1 $y=2x$ and $y=2x+1$ are parallel lines. $y=2x$ and $y=-\frac{1}{2}x-1$ are perpendicular lines.



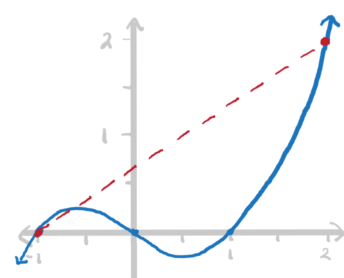
Def The **average rate of change** between any two points is the slope of the line containing the two points. This line is called the **secant line.**

Note: We <u>do not</u> calculate the average rate of change by finding an average. It is <u>not</u> the average of two points.

The average rate of change of a function f from x_1 to x_2 is given by $\frac{f(x_2)-f(x_1)}{x_2-x_1}$.

Ex 2 Let $f(x)=\frac{1}{3}x^3-\frac{1}{3}x$

Rate of change from $(-1,0)$ to $(2,2)$ is $\frac{2-0}{2-(-1)}=\frac{2}{3}$.



Section 1.6

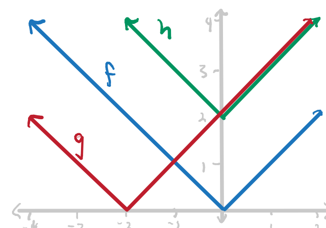
Def A parent function is one without any sort of transformation, like $f(x)=x$, $f(x)=|x|$, $f(x)=x^2$, and so on.

Transformations can affect a function vertically or horizontally.

- Horizontal transformations occur when we modify the function's input
- Vertical transformations occur when we modify the function's output.

Def A shift occurs when we add/subtract a constant.

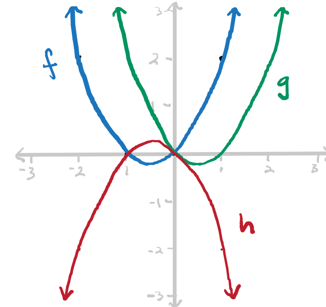
Ex Consider the parent function $f(x)=|x|$. $g(x)=f(x+2)=|x+2|$ shifts the graph left by 2 units. $h(x)=f(x)+2=|x|+2$ shifts the graph up by 2 units.



In short, for a function $f(x)$ and $c>0$,
$f(x+c)$ shifts left, $f(x-c)$ shifts right,
$f(x)+c$ shifts up, $f(x)-c$ shifts down.

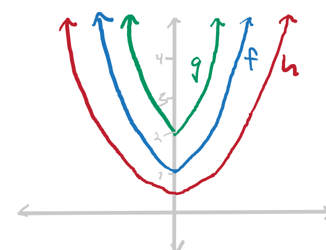
Def A reflection occurs when we multiply by -1.

Ex 3 CONSIDER THE PARENT FUNCTION $f(x) = x^2 + x$. THEN $g(x) = f(-x) = x^2 - x$ IS A REFLECTION ABOUT THE y-AXIS AND $h(x) = -f(x) = -x^2 - x$ IS A REFLECTION ABOUT THE x-AXIS.

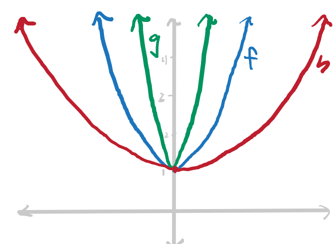


DEF A STRETCH OR SHRINK OCCURS WHEN WE MULTIPLY BY A POSITIVE CONSTANT $c \neq 1$.

Ex 4 (VERTICAL STRETCH / SHRINK) CONSIDER THE PARENT FUNCTION $f(x) = x^2 + 1$. THEN $g(x) = 2f(x) = 2x^2 + 2$ IS A VERTICAL STRETCH AND $h(x) = \frac{1}{2}f(x) = \frac{1}{2}x^2 + \frac{1}{2}$ IS A VERTICAL SHRINK.



Ex 5 (HORIZONTAL STRECH / SHRINK) CONSIDER THE PARENT FUNCTION $f(x) = x^2 + 1$ THEN $g(x) = f(2x) = 4x^2 + 1$ IS A HORIZONTAL SHRINK AND $h(x) = f(\frac{1}{2}x) = \frac{1}{4}x^2 + 1$ IS A HORIZONTAL STRETCH.



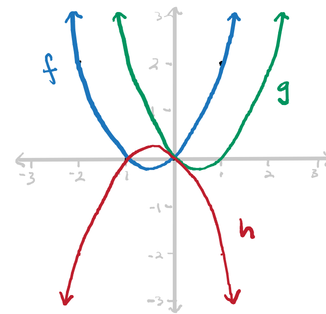
IN SHORT, FOR A FUNCTION f AND A CONSTANT c,

WHEN $c > 1$, $cf(x)$ IS A VERTICAL STRETCH AND $f(cx)$ IS A HORIZONTAL SHRINK.

WHEN $0 < c < 1$, $cf(x)$ IS A VERTICAL SHRINK AND $f(cx)$ IS A HORIZONTAL STRETCH.

WHEN GRAPHING A FUNCTION WITH A SEQUENCE OF TRANSFORMATIONS, PREFORM THEM IN THE FOLLOWING ORDER: 1) HORIZONTAL SHIFT, 2) REFLECTION, 3) STRETCHING/SHRINKING, AND 4) VERTICAL SHIFT.

Ex 3 Consider the parent function $f(x)=x^2+x$. Then $g(x)=f(-x)=x^2-x$ is a reflection about the y-axis and $h(x)=-f(x)=-x^2-x$ is a reflection about the x-axis.



Def A stretch or shrink occurs when we multiply by a positive constant $c \neq 1$.

Ex 4 (Vertical Stretch / Shrink) Consider the parent function $f(x)=x^2$. Then $g(x)=2f(x)=2x^2$ is a vertical stretch and $h(x)=\frac{1}{2}f(x)=\frac{1}{2}x^2$ is a vertical shrink.

Ex 5 (Horizontal Strech / Shrink) Consider the parent function $f(x)=x^3$. Then $g(x)=f(2x)=8x^3$ is a horizontal shrink and $h(x)=f(\frac{1}{2}x)=\frac{1}{8}x^3$ is a horizontal stretch.

In short, for a function f and a constant c,

When $c>1$, $cf(x)$ is a vertical stretch and $f(cx)$ is a horizontal shrink.

When $0<c<1$, $cf(x)$ is a vertical shrink and $f(cx)$ is a horizontal stretch.

When graphing a function with a sequence of transformations, preform them in the following order: 1) Horizontal shift, 2) Reflection, 3) Stretching/shrinking, and 4) Vertical shift.

Ex 6 Say we want to graph the function $p(t) = \frac{1}{3}(t-5)^2 + 2$. Start with the parent function $f(t) = t^2$. Then graph $g(t) = (t-5)^2$, the horizontal shift right. Then graph $h(t) = \frac{1}{3}(t-5)^2$, the vertical shrink. Then graph $p(t) = \frac{1}{3}(t-5)^2 + 2$, the vertical shift.