Section 1.1

Def A relationship between two quantities can be expressed as an equation in two variables. A solution of an equation in two variables, x and y, is an ordered pair (x,y) such that when we substitute these into the equation, we get a true statement.

Ex 1 Consider the equation $y=6x-2$, and the ordered pairs $(1,4)$ and $(3,0)$. If we substitute these values for x and y in our equation, we get:

$$(4)=6(1)-2 \qquad \text{and} \qquad (0)=6(3)-2$$
$$4=4 \qquad\qquad\qquad 0=16$$

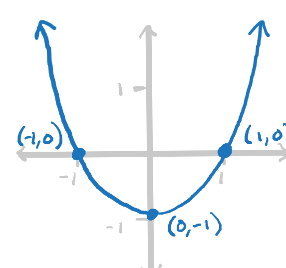
So, $(1,4)$ is a solution of the equation and $(3,0)$ is not.

Def The graph of an equation of two variables is the set of all ordered pairs that "satisfy" (ie are solutions of) the equation.

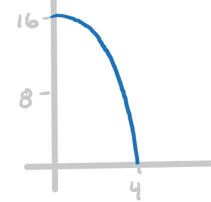
Def An x-intercept is where the graph intersects the x-axis. Note that the y-coordinate of an x-intercept is always zero, so an x-intercept always has the form $(\_,0)$.

Def A y-intercept is where the graph intersects the y-axis. Note that the x-coordinate of a y-intercept is always zero, so a y-intercept always has the form $(0,\_)$.

Ex 2 Let $y=x^2-1$. We see from the graph that the x-intercepts are $(-1,0)$ and $(1,0)$, and the y-intercept is $(0,-1)$.



Ex 3 Suppose $y = 16 - x^2$ represents the height of a ball x seconds after being dropped from a height of 16 ft. The y-intercept is $(0, 16)$ and represents the height of the ball before it is dropped. An x-intercept is $(4, 0)$ and represents the time at which the ball hits the ground, which is 4 seconds after being dropped.



Section 1.2

Def A relation is a set of ordered pairs.

Ex 4 The set $\{(1,0), (13,5), (7,2)\}$ is a relation.

Ex 5 The set $\{(x,y) \mid y = 2x+1\}$ (read "the set of all ordered pairs (x,y) where $y = 2x+1$") is a relation.

Def Given a relation, the set of all first components of the ordered pairs is called the domain. The set of all second components is called the range.

Ex 6 From Example 4, the domain is $\{1, 13, 7\}$ and the range is $\{0, 5, 2\}$.

Ex 7 From Example 5, the domain is $\mathbb{R}$ ("the set of all real numbers"), and the range is $\mathbb{R}$.

Def A function is a correspondence from the domain to the range, such that each element of the domain corresponds to exactly one element of the range.

What the definition of a function is getting at is that we are mapping/sending elements of the domain to elements of the range. Moreover, we require that elements of the domain are sent to <u>exactly one</u> element in the range. (Each input has only one output)

Ex 8 $\{(1,2),(3,4),(5,6),(5,8)\}$ is not a function because 5 (in the domain) is sent to both 6 and 8 (in the range)

Ex 9 $\{(1,2),(3,4),(6,5),(8,5)\}$ is a function because each element of the domain corresponds to only one element of the range.

Note Example 9 demonstrates that a function can have different elements of the domain map to the same element of the range.

In this class, we will be mostly concerned with expressing functions as equations. If we see "y as a function of x" or "y in terms of x," we mean $y=$(some expression involving x).

Ex 10 Let $w=14d-7$. w is a function of d since there is exactly one w-value corresponding to each d-value.

Ex 11 Let $y^2=x^2+1$. Taking square roots of both sides gives us $y=\pm\sqrt{x^2+1}$, each value of x corresponds to 2 values of y, y is not a function of x.

A function can be thought of as a kind of machine, call it f, taking inputs x from the domain and outputting $f(x)$ in the range

Ex 12 Let $f(x) = x^3 - 5$, read as "f of x equals $x^3 - 5$." For each input x, $f(x)$ is the value of the function at x. If, say, we wanted to find $f(1)$ and $f(5)$, we simply substitute in for x in the right-hand-side of the equation.

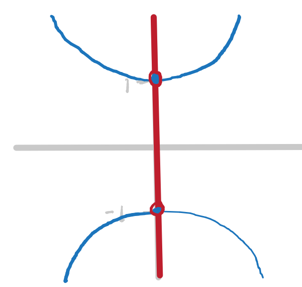
$$f(1) = (1)^3 - 5 = -4, \qquad f(5) = (5)^3 - 5 = 120.$$

This is called **function notation**.

Ex 13 Suppose f, as in Example 12, represents the population of Tempe x years after 1990. Then $f(1)$ is the population in 1991, and $f(5)$ is the population in 1995. If b is some other variable, then $f(b+3)$ is the population b years after 1993.

Def A **graph of a function** f is the set of ordered pairs $(x, f(x))$ plotted on the Cartesian plane. A graph is <u>not a function</u>, but rather a pictorial representation of a function.

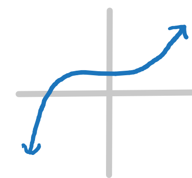
Ex 14 (Vertical Line Test) Let's plot $y^2 = x^2 + 1$ from Example 11. We already determined this was not a function, but we can also see this from the graph. At $x = 0$, $y = \pm 1$, so the vertical line $x = 0$ intersects the graph in two places. This leads to the following result.



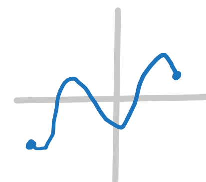
<u>**Vertical Line Test**</u> If any vertical line intersects a graph in more than one point, the graph does not define y as a function of f.

When we look at the graph of a function,

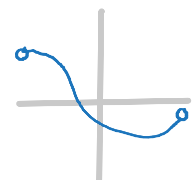
- Arrows indicate that the graph extends indefinitely in the direction of the arrows.



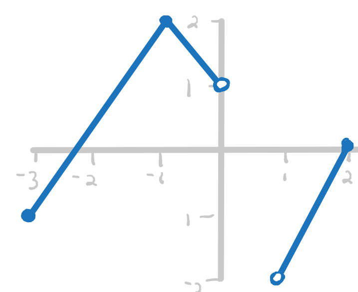
- A closed dot indicates that the graph does not extend past the point and it **includes** that point



- An open circle indicates that the graph does not extend past the point and it **excludes** that point.



Ex 14 Using the graph on the right, we see that the function has



	Interval notation	Set-builder notation
Domain	$[-3,0) \cup (1,2]$	$\{x \mid -3 \le x < 0 \text{ or } 1 < x \le 2\}$
Range	$(-2,2]$	$\{y \mid -2 < y \le 2\}$

Def The zeroes of a function f are the x-values for which $f(x)=0$.

When looking at the graph of a function f, zeroes of a function have the form $(x, 0)$, which means they correspond to x-intercepts. Thus we can find the x-intercepts of a function's graph without ever graphing it.

Since y-intercepts of a graph have the form $(0, \_)$, the y-intercept of the graph of a function is the point $(0, f(0))$.